FINAL REPORT

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Loss of Chaos Synchronization in Presence of a Small Noise or Parameter Mismatch,
and
Bifurcations and Dynamical Transitions in The Quasiperiodically Forced Systems

1. Brief Summary of The Research Work

The research work has two main directions:

1. Loss of chaos synchronization in presence of a small noise or parameter mismatch.
2. Bifurcations and dynamical transitions in the quasiperiodically forced systems.

1.1 Loss of chaos synchronization in presence of a small noise or parameter mismatch

In recent years, synchronization in coupled chaotic systems has become a field of intensive study. Synchronous chaotic attractor (SCA) may exist on some invariant manifold of the phase space. The manifold is asymptotically stable in transverse direction. Destruction of chaotic synchronization is associated with arising of regions of local transversal instability on the manifold. Such an instability occurs due to bifurcations of unstable periodic orbits embedded into the SCA. In vicinities of the transversally unstable cycles the chaotic trajectories near the diagonal undergo exponential divergence from the manifold of synchronous dynamics. Their further evolution depends upon existence/absence of the *absorbing area* near the SCA. The trajectories may return into vicinity of the manifold, or leave it for another attractor of the system or for infinity. In the first case, which is usually referred to as *bubbling transition*, the regime of *weak synchronization* is observed in the system. The dynamics consists of laminar phases and intermittent bursts. The laminar phases correspond to the periods when the trajectory stays near the synchronous manifold, while at the bursting phase the trajectory visits other regions of the phase space. In the case of the second scenario of loss of synchronization (which is referred to as *riddling transition*) the dynamics is reduced to chaotic transient, and trajectories go to another attractor.

We investigate the effect of noise or small parameter mismatch, which are inevitable in natural experiment, on the process of loss of chaos synchronization. In regions of local transversal instability the chaotic trajectories undergo repulsion from the manifold of synchronous dynamics. The strength of repulsion is characterized by positive values of *local (finite-time) Lyapunov exponents*. Due to the positive local transversal exponents, the system demonstrates sensitivity to any parameter mismatch or noise level. Investigating the dynamics of intermittency and chaotic transients, we are interested in distributions of *intervals between bursts* (for bubbling transition) or *lifetime of chaotic transient process* (for riddling transition). The most simple quantities of interest are the *average interburst interval* and the *average lifetime of the chaotic transient*. Refs.[1-2] present some power laws, which give the dependence of the average length of laminar phase (or the average escape time) upon the value of the parameter mismatch or the noise intensity. These results are based on natural and numerical experiments. The aim of our work is to characterize quantitatively the effect of the local transversal instability of the synchronous manifold upon the dynamics of intermittent and transient processes. For this purpose we represent a quantifier, which is determined by properties of local transversal Lyapunov exponents, and which characterizes the distribution laws for the average interburst intervals and the average lifetimes of the chaotic transient. The quantifier can be obtained from local transversal instability analysis of an ensemble of synchronous chaotic orbits. Note, that we consider the properties of *unperturbed* synchronous chaotic trajectories. Thus, we show that characteristics of the intermittency and chaotic transient in presence of noise or parameter mismatch can be obtained from direct analysis of the unperturbed system, without adding a real noise or parameter mismatch. Next, we introduce a representative theoretical model, which gives explanation to statistical laws of intermittent dynamics, observed numerically. The model is based on bounded 1-D random walk process. It describes qualitatively the process of loss of chaos synchronization in wide range of the controlling parameters of the system.

1.2 Bifurcations and dynamical transitions in the quasiperiodically forced systems.

We investigate the behavior of period-doubling dynamical systems under external quasiperiodic forcing. For the case of such systems, numerical analysis and physical experiment show that addition of two-frequency external perturbation gives rise to some bifurcations and crisis phenomena, which are identical in the systems of different nature. That is why as the examples we consider the most simple and well-known dynamical systems: the quasiperiodically forced logistic map (noninvertible system) and Henon map (invertible system). There is a considerable number of publications, which are devoted to phenomenological analysis of bifurcations and crises phenomena observed in these models. However, in many cases the mechanisms of such phenomena remains not quite clear. The aim of our work is to give the detailed explanation for mechanisms of bifurcations and dynamical transitions, which were previously observed. The bifurcations of invariant tori lead to arising of more complicated invariant curves and strange nonchaotic attractors (SNA). Dynamical transitions lead to sudden changes of the shape and number of bands of the attractor, or to disappearance of the attractor.

As the tool of investigation we use the methods of *rational approximations* (RAs) [4] and *critical curves* [5]. In terms of RAs we can give explanation to dynamical transitions for strange nonchaotic (SNA) and chaotic attractors (CA) of the model systems, using well-known mechanisms from dynamics of autonomous and periodically forced systems. By dynamical transitions here we mean the band-merging and basin boundary crises, as well as different types of the interior “widening” crises. The method of critical curves makes it possible to determine the moments of some crises in the parameter space of the system, as well as to localize the attractor in the phase space, and to explain the “shape” of the attractors after crises.

2. Brief Explanation of The Results Obtained During The Visiting Period

2.1 Loss of chaos synchronization in presence of a small noise or parameter mismatch

We introduce a new quantifier, called the noise sensitivity exponent (NSE), which quantitatively characterizes the sensitivity of weak synchronous chaotic attractor to external noise signal. The value of the NSE can be obtained numerically from linear evolution of transverse perturbations of synchronous chaotic trajectories. In terms of the NSE we characterize the effect of noise on intermittent bursting and basin riddling. We show that the average interburst interval and average lifetime of the chaotic transient can be obtained from the NSE. For illustration of the suggested method we considered a system of coupled 1-D maps under influence of (a) additive and (b) parametric noise signal. We obtain the values of NSE for two types of noise: uniformly distributed (bounded noise) and Gaussian (unbounded noise). The values of the NSE appear to be equal in the both cases, and they coincide with the value of parameter sensitivity exponent (PSE), introduced formerly [3]. We give a theoretical explanation to the last result.

We introduce a theoretical model, which explains qualitatively the numerical results for parameter and noise sensitivity exponents. The model is based on bounded 1-D random walk process. Analysis of the model makes it possible to obtain theoretically the asymptotical power laws, which were obtained numerically for the average interburst interval and the average lifetime of the chaotic transient. Results for the random walk model show the increase tendency for the values of the PSE and NSE, as the coupling parameter varies from the first transversal bifurcation and the transversal repulsion becomes more and more strong. The values of PSE and NSE tends to infinity, as we approach the critical value, where blowout bifurcation occurs and the synchronous attractor becomes transversally unstable. These results are in a good qualitative correspondence with the numerical results. In future we hope to obtain quantitative correspondence by determining the values of the PSE and NSE from macroscopic characteristics of distributions of focal Lyapunov exponents.

2.2 Bifurcations and dynamical transitions in the quasiperiodically forced systems

The following results are obtained for the quasiperiodically forced logistic map.

We investigate the mechanisms of the interior “widening” crises of SNAs and CAs. We differ two cases of interior “widening” crises. In the case of multi-band attractors the interior crises leads to sudden change of the attractor into single-band object, and increase of the area, which contains an attractor. For the case when attractor is initially 1-band, the effect of crises consists in the sudden widening of the attractor. Note, that the type of the attractor (SNA or CA) does not change through the crises. An investigation of the mechanism of phenomena in terms of rational approximations shows that in the both cases crises occurs via collision of the chaotic component of approximation with unstable periodic orbit *within* the basin of attraction. Note however, that this mechanism is different from the its analog from periodic dynamics, since the unstable orbit, which collides with the chaotic component, exists only for some values of the phase variable. Thus, there is no smooth limit for unstable orbits as we increase the order of approximation, and the crises mechanism must me regarded as phase-depended. The shape of the attractor after crises is determined by configuration of the critical curves of the map. Note, that crises “simplifies” the shape of the attractor, and the attractor after crises is bounded by less number of critical curves, than before crises.

For the case of band-merging crises, the mechanism is similar to described above. However, in this case there is a smooth quasiperiodic limit for unstable orbits, which collides with chaotic component of the approximation. Hence, we can claim that these crises occur via collision of the attractor with unstable torus.

For the basin boundary crises, destruction of the attractor occurs via collision with unstable torus on boundary of the basin of attraction. In terms of critical curves, this mechanism can be explained as the tangency of the 2-order critical curve with unstable torus.

The method of critical curves gives us an important concept of the *absorbing area*, which contains an attractor. The role of the absorbing area can be illustrated using intermittent scenario of the birth of SNA in the model system. We show, that only in presence of the absorbing area intermittency leads to arising of the SNA and further transition to chaos. In the case, when the absorbing area does not exist, intermittent transition leads to divergence of the trajectories to infinity.

3. Future Plan to Complete The Work and Publication Plan

3.1 Loss of chaos synchronization in presence of a small noise or parameter mismatch

In future we intend to investigate the influence of the unbounded noise with certain asymptotical properties of distributions upon dynamics of intermittent bursts and chaotic transient process. Also we plan to obtain an exact quantitative correspondence between the results, obtained from theoretical model for parameter and noise sensitivity exponents, and the results of numerical experiment. Finally, we hope to determine the values of the PSE and NSE immediately from macroscopic characteristics of distributions of local Lyapunov exponents.
The results will be submitted to SCI journals within a few months.

3.2 Bifurcations and dynamical transitions in the quasiperiodically forced systems

In future we plan to obtain the global structure of bifurcations and crises phenomena in the quasiperiodically forced Henon map. For this purpose we plan to use the method of rational approximations and analysis of evolution of two-dimensional invariant manifolds, related to stable and unstable invariant curves of the map. We believe, that for the case of invertible systems invariant manifolds play the role, which is analogous to the role of critical curve for the case of noninvertible maps.

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