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Loss of Periodic Synchronization in Unidirectionally Coupled Nonlinear Systems

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We have studied desynchronization of the perfectly synchronous periodic orbits of unidirectionally coupled nonlinear systems with a period doubling route to chaos, such as coupled logistic maps and coupled diode resonator circuits. Two different desynchronization mechanisms of a synchronous periodic attractor of the coupled subsystem have been found: one via period doubling bifurcation and the other via transcritical bifurcation. Following a period doubling route to chaos, the periodic orbit of the response subsystem is chaotic after the desynchronization. We show that our results are useful for understanding loss of chaotic synchronization in these unidirectionally coupled systems.

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I. INTRODUCTION

Recently synchronization in coupled nonlinear dynamical systems has been studied extensively [1–7] since synchronization is a basic phenomenon in nature and has many practical applications [8–10]. Synchronization is often understood as a phenomenon in which two coupled systems exhibit identical oscillations. The first observation of synchronization was reported in two coupled clocks by Huygens in 1665. In that case, the synchronization was indicated by the equal periods of the coupled clocks. Today, synchronization is used in a generalized sense. Synchronization exists in coupled periodic and chaotic systems. Complete or identical synchronization, generalized synchronization, and phase synchronization also exist. Complete synchronization means that the periodic or the chaotic oscillations of the coupled identical systems coincide exactly in time due to the strong interaction of the systems [1,2]. Generalized synchronization is a kind of synchronization where the parameters of the coupled systems do not match. One-to-one smooth mapping exists between oscillations of each subsystem so that knowing the state of one subsystem enables one to know the state of the other subsystem [3]. Phase synchronization is defined as the appearance of a certain relationship between the phases of the coupled systems while the amplitude can remain uncorrelated [4].

Since chaos is sensitive to the initial conditions, synchronization of chaos in coupled nonlinear systems with chaotic uncoupled behavior is a striking behavior. Since the famous paper by Pecora and Caroll [2] revived the study of synchronization, various coupling methods for synchronization and many of the concepts necessary for analyzing synchronization have been developed. When these concepts were applied to model systems and also to experimental systems, many interesting phenomena were found when the synchronous oscillations or orbits lost synchronization. Among them are intermittent bursting [11], on-off intermittency [12], the riddling transition [13], and so on.

In this paper, we report two different mechanisms for loss of periodic synchronization in unidirectionally coupled nonlinear model systems and electronic circuit systems with a period doubling route to chaos. When the coupled periodic systems are in complete synchronization, the synchronous periodic attractor (SPA) of the coupled systems is transversely stable. However, the SPA becomes transversely unstable via a period doubling bifurcation or a transcritical bifurcation as the coupling to the response subsystem increases. After the loss of the periodic synchronization, the SPA becomes chaotic following a period doubling route to chaos. Our work is important because the synchronous chaotic attractor (SCA) for unidirectionally coupled nonlinear systems becomes desynchronized by the same mechanisms. In coupled chaotic systems, the loss of chaotic synchronization is related to the transverse stability of the SCA. Similarly, this transverse stability of the SCA is broken by a

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period doubling bifurcation or a transcritical bifurcation. In coupled chaotic systems, it is difficult to see these bifurcations because there are so many unstable periodic orbits (UPO) embedded in the SCA, but it is relatively easy to see these bifurcations in the SPA.

II. SIMULATIONS AND EXPERIMENTAL RESULTS

We consider unidirectionally coupled one-dimensional logistic maps,

$$x_{n+1} = 1 - A{x_n}^2, \quad (1)$$
$$y_{n+1} = 1 - A{y_n}^2 + c({x_n}^2 - {y_n}^2), \quad (2)$$

where x_n and y_n are variables of the drive and the response subsystems, respectively. The constants A and c are the control parameter and the coupling constant of the subsystems, respectively. Unidirectional coupling means that only the dynamics of the response subsystem is affected by the drive subsystem through the coupling; the reverse does not hold. Two coupling methods are generally used: dissipative and linear coupling. The dissipative coupling is used in Eq. (2) because this type of coupling is ubiquitous in coupled nonlinear systems. Without coupling ($c = 0$), each of the subsystems follows a well-known period doubling route to chaos as A increases. With appropriate coupling ($c \neq 0$), a synchronous periodic attractor (SPA) $x_n = y_n$ appears for a given A. Every SPA lies on a diagonal line so that becomes an invariant line of the periodic synchronization. This invariant line has transverse stability, which means that it is stable when perturbed. Here, unidirectional coupling supplies the perturbation.

Figure 1 is the stability diagram of the SPA of unidirectionally coupled logistic maps written as Eqs. (1) and (2). An uncoupled subsystem experiences chaos at the critical parameter value $A^* = 1.401155...$. For moderate coupling (middle region of the c-axis), the SPA is stable, but it becomes unstable as $|c|$ goes beyond the critical value (solid and dashed lines). The solid lines in the figure are period doubling bifurcation lines, and the dashed lines are transcritical bifurcation lines. The integer p in the diagram is the period of the drive or response subsystem. Beyond the solid lines, the SPA becomes transversely unstable due to a period doubling bifurcation of the response subsystem so that $x_n \neq y_n$. For example, the drive subsystem shows a period 1 orbit, but the response subsystem shows a period 2 orbit at $A = 1.1$ and $c = -3.1$. Beyond the dashed lines, an asynchronous periodic attractor (APA) appears. This APA does not lie on an invariant line, but each subsystem has the same period. For example, the drive and the response subsystems show period 1 orbits at $A = 0.9$ and $c = -3.0$, but the two orbits are different ($x_n \neq y_n$), which is in contrast with the SPA.

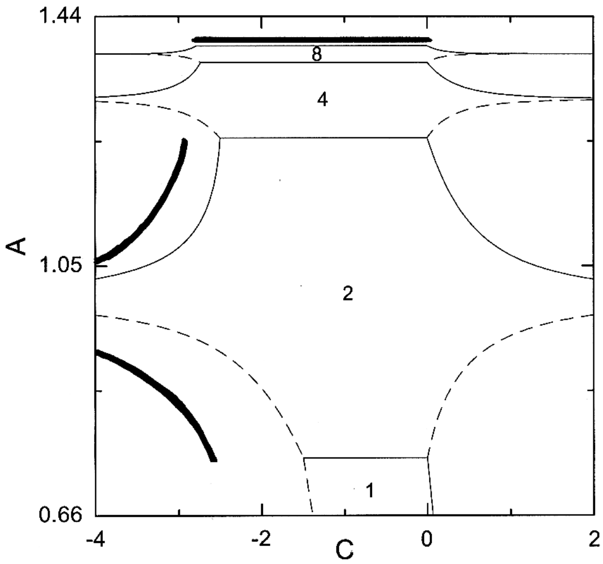

Fig. 1. Stability diagram of the synchronous periodic attractor (SPA) of unidirectionally coupled one-dimensional logistic maps obtained by using a dissipative coupling method. The solid and the dashed lines denote period doubling and transcritical bifurcation lines, respectively. The bifurcation lines for the boundary crisis of the SPA with period 2 are shown as heavy solid lines for reference. The solid horizontal lines are some of infinite sequence of period doubling bifurcations to chaos. The heavy solid horizontal line at the top denotes the Feigenbaum critical line at $A^* = 1.401155...$ The integer p is a period of the orbit of the drive or response subsystem.

Figure 2 clearly shows what is happening after desynchronizing the SPA in these unidirectionally coupled model systems. The SPA desynchronized through a period doubling bifurcation experiences an infinite sequence of period doubling bifurcations, that is, a period doubling route to chaos, as $|c|$ is increasing. This chaotic attractor finally disappears because of a boundary crisis [14]. This means that the unstable APA collides with the chaotic attractor of the coupled systems. The SPA desynchronized through a transcritical bifurcation also experiences an infinite cascade of period doubling bifurcations, so it becomes chaotic as $|c|$ is increased. In this situation, the chaotic attractor which develops from the APA collides with the unstable SPA so it also disappears due to the boundary crisis. The bifurcation lines for the boundary crisis of the SPA with period 2 are shown in Fig. 1 as heavy solid lines for reference. We want to mention the basin of attraction of the SPA of coupled systems. The SPA has a basin of attraction around it since there is a trapping region for uncoupled subsystems. With increasing $|c|$, the basin of the SPA or desynchronized periodic attractor becomes narrower. Outside of the basin of attraction of the SPA, the orbit of the response subsystem grows indefinitely due to the coupling, so the attractor of the coupled systems no longer exists.

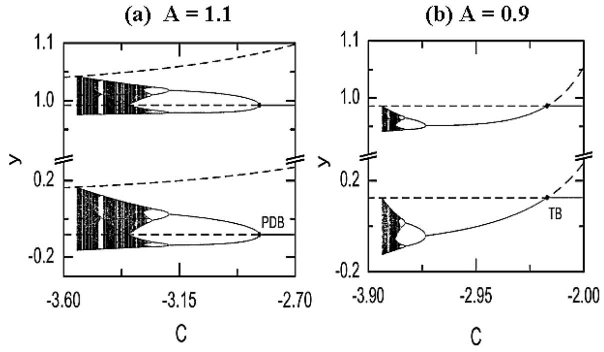

Fig. 2. Bifurcation diagram of the periodic orbit of the response subsystem of unidirectionally coupled one-dimensional logistic maps obtained by using a dissipative coupling method. At $A = 1.1$, the periodic orbit synchronized with that of the drive subsystem is desynchronized at about $c = -2.85$ through a period doubling bifurcation. On the other hand, at $A = 0.9$, the synchronous periodic orbit is desynchronized at about $c = -2.3$ with the drive subsystem through a transcritical bifurcation. For both cases, the asynchronous periodic attractor (APA) of the coupled system experiences an infinite sequence of period doubling bifurcations, causing chaos. Finally, the chaotic attractor disappears through the boundary crisis.

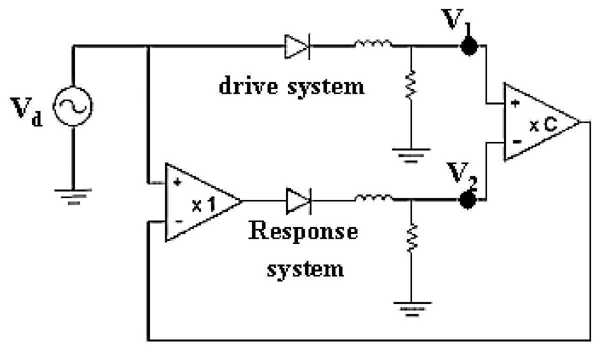

Fig. 3. Schematic circuit diagram of unidirectionally coupled driven diode resonators. The voltage difference of the resistors, $V_1 - V_2$, feedbacks to the response subsystem for unidirectional coupling.

We compare the simulation results of the coupled model systems with the coupled experimental systems. For the experiment, we unidirectionally coupled two driven diode resonator circuits, which are almost identical. Each uncoupled system was well known to show a period doubling route to chaos [15,16] so that it was a good candidate for simulating the one-dimensional logistic map. The unidirectionally coupled diode resonator circuits are very convenient to implement, and it is easy to control the parameters of the subsystems and the coupling constant. The schematic diagram of the unidirectionally coupled diode resonator is shown in Fig. 3. A precise function generator provides a driving force V_d, which is the control parameter of the subsystems. IN4007 Si $p - n$ junction diodes, 10-mH inductors, and 100-Ω resistors are used for the circuits. For the coupled diode resonator circuits, it is generally believed that linear coupling of the output voltage difference of the resistors, $c(V_1 - V_2)$, provides dissipative coupling in coupled model systems. The driving voltage supplied from the function generator was varied from 1.4 V to 2.3 V, and the coupling constant c (amplification factor of the operational amplifier for unidirectional coupling) was changed from -10 to 70.

Figure 4 shows a stability diagram of the SPA measured from unidirectionally coupled diode resonators. (Pn) designates the stability region of an SPA of period n. The synchronous periodic attractor of the coupled circuit becomes transversely unstable because of a period doubling or transcritical bifurcation when the coupling constant c crosses the dashed line. Across the solid line in the figure, the asynchronous periodic or chaotic attractor experiences the boundary crisis so that the size of the attractor suddenly expands indefinitely. Compared to Fig. 1 obtained from the simulation of the unidirectionally coupled logistic maps, Fig. 4 from the experiment has similarities and differences. They are similar in the sense that there is a broad central stability region in the diagram. They are different in the sense that there are no side wings in Fig. 4. Contrary to the one-dimensional logistic map, the driven diode resonator is actually a high-dimensional dynamical system represented by a high-order differential equation [16]. We guess that this high dimensional nature and noise present

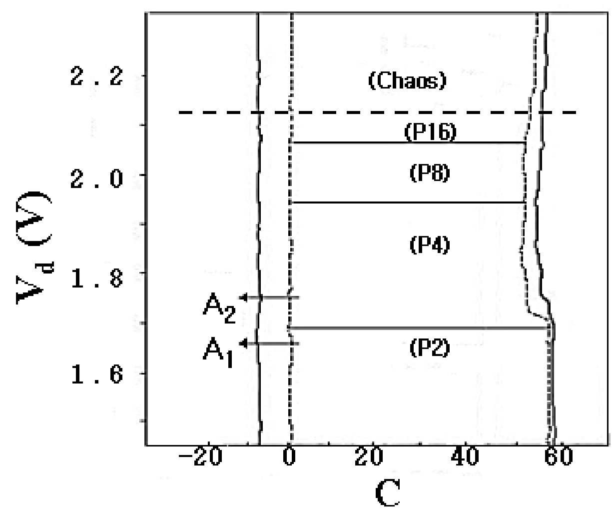

Fig. 4. Stability diagram of the SPA measured from unidirectionally coupled diode resonators. There is a relatively large stability region in the middle of the c-axis, so it is very similar to Fig. 1. On the other hand, the side wings shown in Fig. 1 do not appear here. (Pn) denotes period n of the SPA. Across the dashed lines, the SPA is desynchronized by a period doubling or a transcritical bifurcation. The solid line indicates occurrence of the boundary crisis.

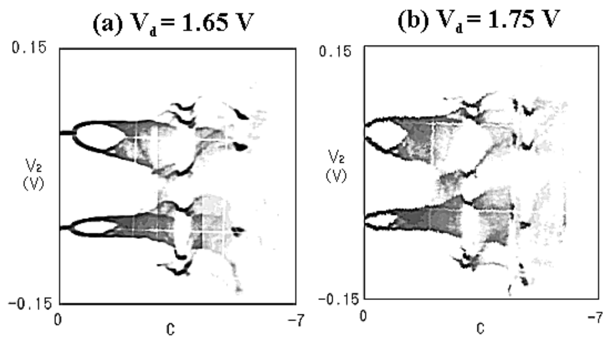

Fig. 5. Bifurcation diagram of the periodic orbit of the response subsystem measured from unidirectionally coupled diode resonators: (a) V_d =1.65 V (along the line A_1 in Fig. 4) and (b) V_d = 1.75 V (along the line A_2 in Fig. 4). We can see that at about $c = -6$, the APA of the coupled systems bursts due to the boundary crisis.

in the circuit may cause the difference in the stability diagrams.

To test the simulation results on mechanisms of loss of the periodic synchronization, we changed the coupling constant of the response circuit following the horizontal lines A_1 and A_2 in Fig. 4. The control parameter were V_d = 1.65 V (period 2) and 1.75 V (period 4) for the line A_1 and A_2, respectively. The bifurcation diagrams of the periodic orbit of the response subsystem for each case are shown in Fig. 5. As seen in Fig. 2, the periodic orbits exhibit infinite sequences of period doubling bifurcations with increasing $|c|$ and are finally burst by the boundary crisis. It is not still clear in Fig. 5 whether the desynchronization of the SPA is due to period doubling bifurcation or due to trancritical bifurcation. Poincare maps of the attractor of the coupled systems might help to clarify the mechanism of desynchronization. In that experiment, the Poincare maps were reconstructed using V_1 and V_2 data sampled with the driving frequency of the function generator.

Figure 6 shows the Poincare maps of the synchronous, asynchronous, chaotic, and bursted attractors of the coupled systems for increasing coupling constant $|c|$. At V_d = 1.65 V, we can clearly see that the SPA becomes desynchronized through a period doubling bifurcation. The SPA is on a diagonal line at $c = 0$ so that it is transversely stable. At $c = -1.4$, it is transversely unstable due to period doubling bifurcation, so dots in the map become separated vertically . On the other hand, at V_d = 1.75 V, the asynchronous periodic attractor ($c = -0.7$) arises from the SPA ($c = 0$) through a trancritical bifurcation. The map of the APA having the same number of dots as that of the SPA is not on a diagonal line, and the dots on the map are not separated vertically. This is a strong indication of the occurrence of a transcritical bifurcation. For both cases, we can see that the loss of synchronization of the SPA is followed by a period doubling route to chaos and the boundary crisis.

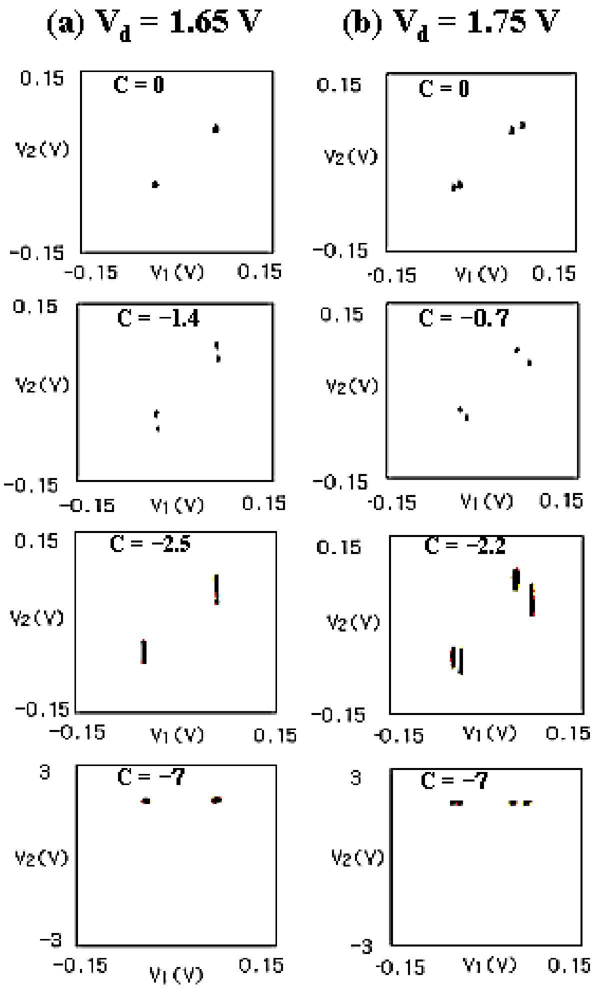

Fig. 6. Poincare maps of the synchronous, asynchronous, chaotic, and bursted attractor of the unidirectionally coupled diode resonators at various c. (a) At V_d =1.65 V, the SPA is desynchronized by a period doubling bifurcation at about $c = -1.4$. The dots are separated vertically. (b) At V_d =1.75 V, the SPA is desynchronized by a transcritical bifurcation at about $c = -0.7$. The dots of the APA are not on the diagonal line, but the number of the dots is the same as that of the SPA.

III. CONCLUSIONS

We have shown by simulation and experiment that loss of periodic synchronization of unidirectionally coupled nonlinear dynamical systems with a period doubling route to chaos is caused by two different types of bifurcation: period doubling and transcritical bifurcations. When dissipative coupling is used, a relatively large stability region in the middle range of the coupling constant c is found, which means that dissipative coupling is effective for maintaining the SPA. Although the results for linear coupling in unidirectionally coupled systems are not reported here, they show that the stability region of the SPA is much narrower than that for dissipative coupling. Also, the stability region for linear coupling shows complicated tree structures [17]. We have tested the periodic synchronization of globally or bidirectionally coupled nonlinear systems as well. Bidirectional coupling is found to disturb the synchronization, instead of helping it, so that the stability region is reduced almost by half comparing to that of unidirectional coupling. This means that the stability of the SPA or the SCA of the coupled systems depends in a crucial way on the coupling method.

In the context of the periodic synchronization investigated in this paper, we can conclude that chaos synchronization can also be achieved most efficiently by using unidirectional coupling of dynamical systems with a dissipative coupling method. Therefore, our results are also useful for guiding efficient chaotic synchronization and for understanding the loss of chaotic synchronization in coupled dynamical systems.

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