# Characterization of Stochastic Spiking Coherence in Coupled Neurons

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We consider a large population of globally coupled subthreshold Morris-Lecar neurons. By varying the noise intensity D, we investigate numerically stochastic spiking coherence (*i.e.*, noise-induced coherence between neural spikings). As D passes a threshold, a transition from an incoherent to a coherent state occurs. This coherent transition is described in terms of the "thermodynamic" order parameter  $\mathcal{O}$ , which concerns a macroscopic time-averaged fluctuation of the global potential. We note that such stochastic spiking coherence may be well visualized in terms of the raster plot of neural spikings (*i.e.*, spatiotemporal plot of neural spikings), which is directly obtained in experiments. To quantitatively measure the degree of stochastic spiking coherence (seen in the raster plot), we introduce a new type of "spiking coherence measure,"  $M_s$ , by taking into consideration the average contribution of (microscopic) local neural spikings to the (macroscopic) global membrane potential. Hence, the spiking coherence measure may be regarded as a "statistical-mechanical" measure. Through competition between the constructive and the destructive roles of noise, stochastic spiking coherence is found to occur over a large range of intermediate noise intensities and to be well characterized in terms of the mutually complementary quantities of  $\mathcal{O}$  and  $M_s$ . Particularly,  $M_s$ reflects the degree of stochastic spiking coherence seen in the raster plot very well.

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### I. INTRODUCTION

In recent years, much attention has been paid to brain rhythms [1]. Synchronization of the firing activity of groups of neurons may be used for efficient sensory processing (e.g., visual binding) [2]. In addition to a constructive role of encoding sensory stimuli, neural synchronization is also correlated with pathological rhythms associated with neural diseases (e.g., epilepticseizures and tremors in Parkinson's disease) [3]. To understand the mechanisms of synchronized firings, collective dynamical behaviors have been investigated in coupled systems composed of spontaneously firing (*i.e.*, selfoscillating) neurons. As a result of these studies, three mechanisms for neural synchronization have been found [4]. In this paper, we are interested in noise-induced coherent dynamics in neural networks consisting of subthreshold neurons. (Each subthreshold neuron in the absence of coupling cannot fire spontaneously without noise; it can fire only with the help of noise.) A main subject of our study is to investigate collective coherence between noise-induced firings, which is in contrast to studies of synchronization between spontaneous (selfsustained) firings. Noise is usually considered as a nuisance, degrading the performance of dynamical systems.

This paper is organized as follows. In Sec. II, we consider a large population of globally coupled subthreshold Morris-Lecar (ML) neurons [8–10]. By varying the noise intensity D, we investigate numerically stochastic spiking coherence (*i.e.*, noise-induced coherence between neural spikings) for a fixed coupling strength. For small D, neurons fire independently; hence, the global output signal (*i.e.*, the ensemble-averaged membrane potential) becomes incoherent. However, as the noise amplitude passes a threshold, the global output signal becomes coherent (*i.e.*, it exhibits a collective motion on a noisy limit cycle). As in globally coupled chaotic systems [11– 13], this kind of coherent transition may be described in terms of an order parameter  $\mathcal{O}$ . For our case, the mean square deviation of the global output signal plays the role of  $\mathcal{O}$ , which is a "thermodynamic" quantity because it concerns a macroscopic time-averaged fluctuation of the global potential. Such stochastic spiking coherence may be well visualized in terms of the raster plot of neural spikings (*i.e.*, a spatiotemporal plot of neural spikings) which is directly obtained in experiments. To

However, in certain circumstances, noise plays a constructive role in the emergence of dynamical order. A distinguished example of such manifestations is the stochastic resonance through which noise-enhanced detection of a weak signal may occur [5,6]. Recently, there has been great interest in noise-induced coherence in coupled excitable systems [7].

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quantitatively measure the degree of stochastic spiking coherence (seen in the raster plot), we introduce a new type of coherence measure,  $M_s$ , called the spiking coherence measure, by considering the average contribution of (microscopic) local spikings to the (macroscopic) global membrane potential. Hence,  $M_s$  may be regarded as a "statistical-mechanical" measure, in contrast to the thermodynamic order parameter  $\mathcal{O}$ . Through competition between the constructive and the destructive roles of noise, stochastic spiking coherence is found to occur over a large range of intermediate noise intensities, and to be well characterized in terms of the mutually complementary quantities of  $\mathcal{O}$  and  $M_s$ . As the noise amplitude is increased from the threshold,  $M_s$  increases dramatically at first, showing onset of coherence, because noise stimulates collective coherence between neural spikings. Then, the values of  $M_s$  become nearly the same over a large range of intermediate D, but for large D,  $M_s$  decreases because noise spoils spiking coherence. In this way,  $M_s$  reflects the degree of stochastic spiking coherence seen in the raster plot very well. Hence, we expect that  $M_s$  may be implemented for characterizing the degree of coherence in the experimentally-obtained raster plot of neural spikings. Finally, a summary is given in Sec. III.

### II. CHARACTERIZATION OF STOCHASTIC SPIKING COHERENCE

We consider a system of N globally coupled neurons. As an element in our coupled system, we choose the conductance-based ML neuron model, originally proposed to describe the time-evolution pattern of the membrane potential for the giant muscle fibers of barnacles [8–10]. The population dynamics in this neural network is governed by the following set of differential equations:

$$C\frac{dv_i}{dt} = -I_{ion,i} + I_{DC} + D\xi_i + I_{syn,i},$$
(1a)

$$\frac{dw_i}{dt} = \phi \frac{(w_{\infty}(v_i) - w_i)}{\tau_R(v_i)}, \quad i = 1, \cdots, N,$$
(1b)

where

$$I_{ion,i} = I_{Ca,i} + I_{K,i} + I_{L,i}$$
(2a)  
- a\_{G} m\_{(v\_{i})}(v\_{i} - E\_{G})

$$-g_{Ca}m_{\infty}(v_{i})(v_{i} - E_{Ca}) + g_{K}w_{i}(v_{i} - E_{K}) + g_{L}(v_{i} - E_{L}),$$
(2b)

$$I_{syn,i} = \frac{J}{N-1} \sum_{\substack{j(\neq i)}}^{N} \Theta(v_j - v^*), \qquad (2c)$$

$$m_{\infty}(v) = 0.5 \left[1 + \tanh\left\{(v - V_1)/V_2\right\}\right],$$
 (2d)

$$w_{\infty}(v) = 0.5 \left[1 + \tanh\left\{(v - V_3)/V_4\right\}\right],$$
(2e)

$$\tau_R(v) = 1/\cosh\left\{(v - V_3)/(2V_4)\right\}.$$
(2f)

Here, the state of the ith neuron at a time t (measured in units of ms) is characterized by two state variables: the membrane potential  $v_i$  (measured in units of mV) and the slow recovery variable  $w_i$  representing the activation of the  $K^+$  current (*i.e.*, the fraction of open  $K^+$ channels). In Eq. (1a), C represents the capacitance of the membrane of each neuron, and the time evolution of  $v_i$  is governed by four kinds of source currents.

The total ionic current  $I_{ion,i}$  of the *i*th neuron consists of the calcium current  $I_{Ca,i}$ , the potassium current  $I_{K,i}$ , and the leakage current  $I_{L,i}$ . Each ionic current obeys Ohm's law. The constants  $g_{Ca}$ ,  $g_K$  and  $g_L$  are the maximum conductances for the ion and the leakage channels, and the constants  $E_{Ca}$ ,  $E_K$  and  $E_L$  are the reversal potentials at which each current is balanced by the ionic concentration difference across the membrane. Since the calcium current  $I_{Ca,i}$  changes much faster than the potassium current  $I_{K,i}$ , the gate variable  $m_i$  for the  $Ca^{2+}$  channel is assumed to always take its saturation value  $m_{\infty}(v_i)$ . On the other hand, the activation variable  $w_i$  for the  $K^+$  channel approaches its saturation value  $w_{\infty}(v_i)$  with a relaxation time  $\tau_R(v_i)/\phi$ , where  $\tau_R$  has a dimension of ms and  $\phi$  is a (dimensionless) temperature-like time scale factor.

Each ML neuron is also stimulated by the common DC current  $I_{DC}$  and an independent Gaussian white noise  $\xi$ [see the second and third terms in Eq. (1a)] satisfying  $\langle \xi_i(t) \rangle = 0$  and  $\langle \xi_i(t) \ \xi_j(t') \rangle = \delta_{ij} \ \delta(t-t')$ , where  $\langle \cdots \rangle$ denotes the ensemble average. The noise  $\xi_i$  is a parametric one and randomly perturbs the strength of the applied current  $I_{DC}$ , and its intensity is controlled by the parameter D. The last term in Eq. (1a) represents the coupling of the network. Each neuron is connected to all the others through global instantaneous pulse-type synaptic couplings.  $I_{syn,i}$  of Eq. (2c) represents such a synaptic current injected into the ith neuron. The coupling strength is controlled by the parameter  $J, \Theta(x)$  is the Heaviside step function (*i.e.*,  $\Theta(x) = 1$  for  $x \ge 0$ and  $\Theta(x) = 0$  for x < 0 and  $v^*$  is the threshold value for the spiking state (*i.e.*, for  $v_i > v^*$ , a local spiking state of the ith neuron appears). Here, we consider the excitatory coupling of J > 0 and set  $v^* = 0$  mV.

The ML neuron may exhibit either type-I or type-II excitability, depending on the system parameters. Throughout this paper, we consider the case of type-II excitability where  $g_{Ca} = 4.4 \text{ mS/cm}^2$ ,  $g_K = 8 \text{ mS/cm}^2$ ,  $g_L = 2 \text{ mS/cm}^2$ ,  $E_{Ca} = 120 \text{ mV}$ ,  $E_K = -84 \text{ mV}$ ,  $E_L = -60 \text{ mV}$ ,  $C = 5 \mu \text{F/cm}^2$ ,  $\phi = 0.04$ ,  $V_1 = -1.2 \text{ mV}, V_2 = 18 \text{ mV}, V_3 = 2 \text{ mV}$  and  $V_4 =$ 30 mV [14]. As  $I_{DC}$  passes a threshold in the absence of noise, each single type-II ML neuron begins to fire with a nonzero frequency that is relatively insensitive to the change in  $I_{DC}$  [15,16]. Numerical integration of Eq. (1) is done using the Heun method [17] (with the time step  $\Delta t = 0.01$  ms) similar to the second-order Runge-Kutta method, and data for  $(v_i, w_i)$  (i = 1, ..., N) are obtained with the sampling time interval  $\Delta t = 1$  ms. For each realization of the stochastic process in Eq. (1), we choose a random initial point  $[v_i(0), w_i(0)]$  for the *i*th (i = 1, ..., N) neuron with uniform probability in the



Fig. 1. Coherent and incoherent states for  $J = 50 \ \mu\text{A/cm}^2$ . Snapshot profiles for a coherent state at (a1) t = 1070 ms and (a2) 1135 ms for  $N = 10^3$  and  $D = 0.3 \ \mu\text{A} \cdot \text{ms}^{1/2}/\text{cm}^2$ . Local states  $(v_i, w_i)$   $(i = 1, ..., 10^3)$  are represented by black dots and the global state  $(V_G, W_G)$  is denoted by a cross. The global state makes a counterclockwise rotation on a noisy gray limit cycle. (b) Time series of  $V_G(t)$  for  $N = 10^3$  and  $D = 0.3 \ \mu\text{A} \cdot \text{ms}^{1/2}/\text{cm}^2$ . Snapshot profiles for an incoherent state at (c1) t = 1070 ms and (c2) 1135 ms for  $N = 10^3$ and  $D = 30 \ \mu\text{A} \cdot \text{ms}^{1/2}/\text{cm}^2$ . Local and global states are represented in the same way as in (a). (d) Time series of  $V_G(t)$  for  $N = 10^3$  and  $D = 30 \ \mu\text{A} \cdot \text{ms}^{1/2}/\text{cm}^2$ . (e) Plots of  $\log_{10} \mathcal{O}$  versus  $\log_{10} D$ .

range of  $v_i(0) \in (-60, 60)$  and  $w_i(0) \in (0.1, 0.5)$ .

We consider a large excitatory population of globally coupled ML neurons for a subthreshold case of  $I_{DC} = 84$  $\mu$ A/cm<sup>2</sup>. For an isolated single case, each subthreshold neuron cannot fire spontaneously in the absence of noise, and it may generate firings only with the aid of noise. We set  $J = 50 \ \mu$ A/cm<sup>2</sup> and numerically investigate stochastic spiking coherence (*i.e.*, collective coherence between noise-induced firings) by varying the noise amplitude D. Emergence of global spiking coherence in the population may be described by the population-averaged membrane potential  $V_G$  (corresponding to the global potential) and the global recovery variable  $W_G$ ,

$$V_G(t) = \frac{1}{N} \sum_{i=1}^{N} v_i(t)$$
 and  $W_G(t) = \frac{1}{N} \sum_{i=1}^{N} w_i(t).$  (3)

For  $N = 10^3$  and  $D = 0.3 \ \mu\text{A} \cdot \text{ms}^{1/2}/\text{cm}^2$ , snapshots for a coherent state are shown in Figures 1(a1) and 1(a2) at t = 1070 ms and 1135 ms, respectively. Local states  $(v_i, w_i)$   $(i = 1, \dots, N)$  are denoted by black dots, and the global state  $(V_G, W_G)$  is represented by a cross. As time goes on, the global state exhibits a counterclock-



Fig. 2. Raster plots of noise-induced spikings (*i*: neuron index and *t*: time) [time series of  $V_G(t)$ ] for D = (a1)-(b1) 0.12, (a2)-(b2) 0.3, (a3)-(b3) 3, (a4)-(b4) 10 and (a5)-(b5) 13  $\mu A \cdot ms^{1/2}/cm^2$  when  $J = 50 \ \mu A/cm^2$  and  $N = 10^3$ .

wise rotation on a noisy gray limit cycle; hence, collective coherent oscillatory motion occurs, as shown in Figure 1(b). On the other hand, Figures 1(c1) and 1(c2) show snapshots of an incoherent state, where neurons fire independently, for  $N = 10^3$  and  $D = 30 \ \mu\text{A} \cdot \text{ms}^{1/2}/\text{cm}^2$ . For this case, the global potential  $V_G$  is nearly stationary [see Figure 1(d)]. As in globally coupled chaotic systems [11–13], the mean square deviation of the global potential  $V_G$ ,

$$\mathcal{O} \equiv (V_G(t) - \overline{V_G(t)})^2, \tag{4}$$

plays the role of the order parameter used for describing the coherence-incoherence transition, where the overbar represents time averaging. Such an order parameter is a "thermodynamic" quantity because it concerns just the macroscopic time-averaged fluctuation of  $V_G$ . Here, we discard the first time steps of a stochastic trajectory as transients for  $10^3$  ms; then, we numerically compute  $\mathcal{O}$ by following the stochastic trajectory for  $10^4$  ms. For the coherent (incoherent) state, the order parameter  ${\cal O}$ approaches a nonzero (zero) limit value in the thermodynamic limit of  $N \to \infty$ . Figure 1(e) shows a plot of the order parameter versus the noise intensity. For  $D < D_l^* (\simeq 0.115 \ \mu \mathrm{A} \cdot \mathrm{ms}^{1/2}/\mathrm{cm}^2)$ , incoherent states exist because the order parameter  $\mathcal{O}$  tends to zero as  $N \to \infty$ . As D passes the lower threshold  $D_1^*$ , a coherent transition occurs because of the constructive role of noise to stimulate coherence between noise-induced spikings. However, for large  $D > D_h^*$  ( $\simeq 16 \ \mu \text{A} \cdot \text{ms}^{1/2}/\text{cm}^2$ ) such coherent states disappear (i.e., a transition to an incoherent state occurs when D passes the higher threshold  $D_h^*$ ) due to the destructive role of noise to spoil the spiking coherence.

Stochastic spiking coherence may be well visualized in terms of the raster plot of neural spikings (*i.e.*, spatiotemporal plot of neural spikings), which is directly obtained in experiments. Figures 2(a1)-2(a5) show such raster plots for  $N = 10^3$  in the coherent region for D = 0.12, 0.3, 3, 10 and  $13 \ \mu \text{A} \cdot \text{ms}^{1/2}/\text{cm}^2$ , respectively, and the time series of their corresponding global



Fig. 3. Spiking coherence measure for  $J = 50 \ \mu \text{A/cm}^2$ and  $N = 10^3$ . Time series of (a) the global potential  $V_G(t)$ and (b) the global phase  $\Phi(t)$  for  $D = 0.3 \ \mu \text{A} \cdot \text{ms}^{1/2}/\text{cm}^2$ . Here, the cross represents the beginning point of each global spiking state. Plots of the spiking coherence measure  $M_i$  of the *i*th neuron for  $D = (c1) \ 0.12$ , (c2) 0.3, (c3) 3, (c4) 10 and (c5) 13  $\mu \text{A} \cdot \text{ms}^{1/2}/\text{cm}^2$ . (d) Plot of the (ensemble-averaged) spiking coherence measure,  $M_s$ , versus  $\log_{10} D$ .

potentials  $V_G$  are also given in Figures 2(b1)-2(b5). For an optimal noise intensity  $D^*$  ( $\simeq 0.3 \ \mu \dot{A} \cdot m s^{1/2} / cm^2$ ), clear stripes (consisting of local neural spikings) appear successively at nearly regular time intervals  $\Delta t ~(\simeq 82)$ ms), as shown in Figure 2(a2). For this optimal case, the degree of stochastic spiking coherence seems to be maximal. As D is increased from  $D^*$ , such an optimal degree of stochastic spiking coherence is nearly preserved over a large range of D [e.g., see Figure 2(a3) for D = 3 $\mu A \cdot ms^{1/2}/cm^2$  because only a little smearing of stripes occurs. We also note that the amplitude of the global potential  $V_G$  for  $D = 3 \ \mu \text{A} \cdot \text{ms}^{1/2}/\text{cm}^2$  is nearly the same as that for  $D = 0.3 \ \mu \text{A} \cdot \text{ms}^{1/2}/\text{cm}^2$ , although the frequency increases with D. However, for large D, stripes in the raster plots become more and more smeared [e.q.,see Figures 2(a4) and 2(a5)] and the amplitude of  $V_G$ decreases [e.q.] see Figures 2(b4) and 2(b5)] because of the destructive role of noise to spoil the stochastic spiking coherence. Hence, the degree of stochastic spiking coherence decreases. In contrast to the case of large D, for small D (less than  $D^*$ ) not only smearing but also skipping of stripes occur in the raster plot; due to skipping the number of stripes in a given time interval decreases. As an example, see Figure 2(a1) for  $D = 0.12 \ \mu \text{A} \cdot \text{ms}^{1/2}/\text{cm}^2$ . Thus, the corresponding global potential becomes irregular, particularly due to skipping, as shown in Figure 2(b1). In this way, the degree of stochastic spiking coherence decreases as D is decreased from  $D^*$ .

To quantitatively measure the degree of stochastic

spiking coherence seen in the raster plot, we first introduce a global phase of the global potential  $V_G(t)$ . Figure 3(a) shows a time series of the global potential for D = $0.3 \ \mu \text{A} \cdot \text{ms}^{1/2}/\text{cm}^2$ . When  $V_G$  is larger (smaller) than a threshold value  $V_G^* [= (V_{G,max} + V_{G,min})/2; V_{G,max(min)}$ is the maximum (minimum) of  $V_G$ ], a spiking (silent) state appears. For the *j*th spiking state of  $V_G$  (*j* =  $1, 2, 3, \ldots$ ), its beginning point at  $t = t_j^*$  is represented by a cross. Thus, a combination of a spiking state (denoted by a black curve) and the following silent state (represented by a gray curve) forms a global cycle G. Two complete global cycles,  $G_1$  and  $G_2$ , are shown in Figure 3(a). Here, the first global cycle  $G_1$  begins from the global spiking state that first appears after a transient process of  $10^3$  ms. It is then possible to introduce an instantaneous global phase  $\Phi(t)$  by using linear interpolation [18],

$$\Phi(t) = 2\pi(j-1) + 2\pi(\frac{t-t_j^*}{t_{j+1}^* - t_j^*}) \quad \text{for}$$
  
$$t_j^* \le t \le t_{j+1}^* \quad (j = 1, 2, 3, \dots).$$
(5)

The global phase  $\Phi(t)$  for  $D = 0.3 \ \mu \text{A} \cdot \text{ms}^{1/2}/\text{cm}^2$  is , thus, obtained, as shown in Figure 3(b). Then, the degree of stochastic spiking coherence (seen in the raster plot) is measured by taking into consideration the average contribution of (microscopic) local spikings to the (macroscopic) global potential  $V_G$ . Hence,  $M_s$  may be regarded as a "statistical-mechanical" measure, which is in contrast to the "thermodynamic" order parameter  $\mathcal{O}$ . We follow 200 global cycles and obtain local spikings. Consider an sth local spiking of the ith neuron beginning at the time  $t_i^{(s)}$ . Then, the contribution of the sth local spiking to  $V_G$  is given by  $\cos \Phi_i^{(s)}$ , where  $\Phi_i^{(s)}$  is just the global phase at the sth local spiking time [*i.e.*,  $\Phi_i^{(s)} \equiv \Phi(t_i^{(s)})$ ]. A local spiking makes the most constructive (in-phase) contribution to  $V_G$  when the corresponding global phase  $\Phi_i^{(s)}$  is  $2\pi n \ (n = 0, 1, 2, ...)$ , while it makes the most destructive (anti-phase) contribution to  $V_G$  when  $\Phi_i^{(s)}$  is  $2\pi(n+1/2)$ . By averaging the contribution of all local spikings, we obtain the spiking coherence measure of the ith neuron,

$$M_{i} = \frac{1}{L_{i}} \sum_{s=1}^{L_{i}} \cos \Phi_{i}^{(s)}, \tag{6}$$

where  $L_i$  is the total number of local spikings of the *i*th neuron. Figures 3(c1)-3(c5) show plots of  $M_i$  versus *i* for D = 0.12, 0.3, 3, 10 and  $13 \ \mu \text{A} \cdot \text{ms}^{1/2}/\text{cm}^2$ , respectively. Fluctuations occur in  $M_i$ . By averaging  $M_i$  over all neurons, we get the (population-averaged) spiking coherence measure  $M_s$ ,

$$M_{s} = \frac{1}{N} \sum_{i=1}^{N} M_{i}.$$
 (7)

For a coherent state,  $0 < M_s \leq 1$ , while for an incoherent state,  $M_s = 0$ . Figure 3(d) shows the plot of  $M_s$  versus

the noise amplitude D. As D is increased from the lower threshold  $D_l^*$ ,  $M_s$  increases abruptly at first, indicating onset of coherence, and then a wide plateau with nearly constant large  $M_s$  is followed. Thus, stochastic spiking coherence with large  $M_s$  becomes stable against a large range of intermediate noise intensity. However, for large  $D M_s$  decreases due to the destructive role of noise to spoil the spiking coherence. In this way, the spiking coherence measure  $M_s$  reflects the degree of stochastic spiking coherence seen in the raster plots well.

# **III. SUMMARY**

We have numerically investigated stochastic spiking coherence in a population of globally coupled subthreshold ML neurons by varying the noise amplitude D. As D passes a threshold, a transition to a coherent state occurs. This coherent transition has been described in terms of the "thermodynamic" order parameter  $\mathcal{O}$ , which concerns just the macroscopic time-averaged fluctuation of the global potential. We note that such stochastic spiking coherence is well visualized in the raster plots of neural spikings which are directly obtained in experiments. To quantitatively measure the degree of stochastic spiking coherence seen in the raster plots, we introduce a new type of "statistical-mechanical" spiking coherence measure  $M_s$  by considering the average contribution of (microscopic) local spikings to the (macroscopic) global potential. Through competition between the constructive and the destructive roles of noise, stochastic spiking coherence is found to occur over a large range of intermediate noise intensity and to be well characterized in terms of the mutually complementary quantities of  $\mathcal{O}$  and  $M_s$ . Particularly,  $M_s$  reflects the degree of stochastic spiking coherence seen in the raster plot very well. Hence, we expect that  $M_s$  may be implemented to characterize the degree of coherence in an experimentally-obtained raster plot of neural spikings. (Such implementation of  $M_s$  for analyzing real experimental data is beyond the present work, and is left as a future work.) We also make some comments on another type of "phase coherence" measure  $M_p$ , which is defined by considering the difference between local phases associated with spikings of neighboring neurons [19].  $M_p$ has often been used to characterize phase coherence in a system of coupled neurons. However, the direct relation between  $M_p$  and stochastic spiking coherence seen in the raster plot is not so clear because  $M_p$  reflects just the average correlation between local phases of neighboring neurons. On the other hand, the newly-introduced  $M_s$ directly reflects the degree of coherence seen in the raster plot. Finally, we note that stochastic spiking coherence might be an origin for synchronous brain rhythms in a noisy environment, which correlate with the brain function of encoding sensory stimuli or are associated with neural diseases.

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