Effect of Spike-Timing-Dependent Plasticity on Stochastic Spike Synchronization in A Small-World Neuronal Network

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- **Stochastic Spike Synchronization (SSS)**

Subthreshold neurons (which cannot fire spontaneously): Firing only with the help of noise.

SSS: Population synchronization between complex noise-induced firings of subthreshold neurons which exhibit irregular discharges like Geiger counters.

Occurrence of SSS in an intermediate range of noise intensity via competition between the constructive and the destructive roles of noise.

Synaptic Plasticity

- **Synaptic Plasticity**
  Adjustments of Synapses: Variation (potentiation or depression) for adaptation to the environment → Synaptic Plasticity
  Basis for learning, Memory, and Development

- **Hebbian spike-timing-dependent plasticity (STDP)**
  Hebbian STDP rule → Variation of synaptic strengths depending on the relative time difference between the pre- and the post-synaptic spike times.

  Pre-synaptic spike precedes a post-synaptic spike → long-term potentiation (LTP) ; Otherwise, long-term depression (LTD)

- **Purpose of Our Study**
  In the previous works on SSS, synaptic coupling strengths are static.

  Investigation of the effect of STDP on the SSS by varying the noise intensity in an excitatory population of subthreshold neurons

Excitatory Small-World Network of Subthreshold Regular Spiking (RS) Izhikevich Neurons with Synaptic Plasticity

- **Small-World Network (SWN) of RS Izhikevich Neurons**

  Watts-Strogatz SWN with the rewiring probability p=0.15 and the average number of synaptic inputs per neuron M_{syn}=20

  Subthreshold RS Neurons with the DC current $I_{DC,i} \in [3.55, 3.65]$

- **Hebbian STDP**

  Update of coupling strengths: Additive nearest-spike pair-based STDP rule

$$J_{ij} \rightarrow J_{ij} + \delta \Delta J_{ij}(\Delta t_{ij})$$

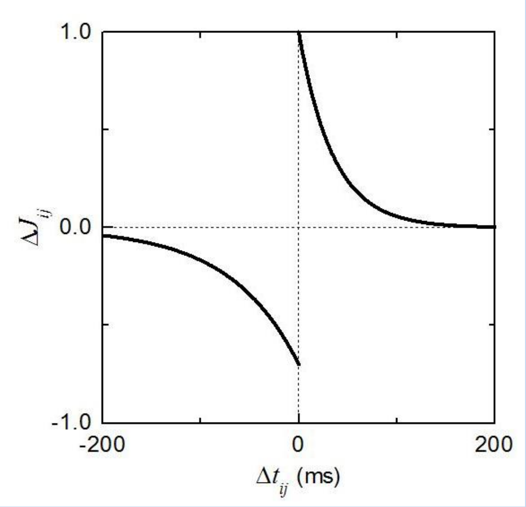
$$\Delta t_{ij} = t_i^{(post)} - t_j^{(pre)}, \;\; \delta = 0.005$$

$$J_{ij} \in [J_l(=0.0001), J_h(=1)]$$

Initial synaptic strengths: Mean J_0=0.2 & standard deviation σ=0.02

Asymmetric time window for ΔJ_{ij}

$$\Delta J_{ij} = \begin{cases} A_+ e^{-\Delta t_{ij}/\tau_+} & \text{for } \Delta t_{ij} > 0 \\ -A_- e^{\Delta t_{ij}/\tau_-} & \text{for } \Delta t_{ij} < 0 \end{cases} \qquad A_+ = 1.0, A_- = 0.7, \;\; \tau_+ = 35\text{ms}, \tau_- = 70\text{ms}$$



$\Delta t_{ij} > 0 \rightarrow$ LTP, $\Delta t_{ij} < 0 \rightarrow$ LTD

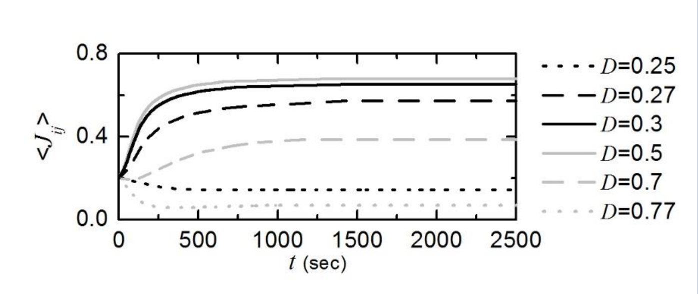
Effect of the STDP on the SSS

- **SSS in the Absence of the STDP**

  Occurrence of SSS in the range of D (D_l^*[~0.225], D_h^*[~0.846]).
  Appearance of SSS when passing D_l^* thanks to a constructive role of noise to stimulate coherence between noise-induced spikings;
  Disappearance of SSS when passing D_h^* due to a destructive role of noise to spoil SSS.

- **Time-Evolution of Population-Averaged Synaptic Strength $<J_{ij}>$**

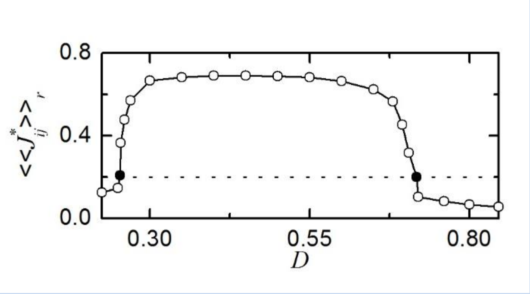
  LTP (D=0.27, 0.3, 0.5, & 0.7): Monotonic Increase in $<J_{ij}>$ above the initial average value J_0 (=0.2) and saturated limit value $< J_{ij}^* >$ nearly at 2000 sec.
  LTD (D=0.25 & 0.77): Monotonic decrease in $<J_{ij}>$ below J_0 and saturated limit value $< J_{ij}^* >$



- **Population-Averaged Limit Values of Synaptic Strengths $<<J_{ij}>>_r$**

  Occurrence of LTP in the range of ($\tilde{D}_l$[~0.253], $\tilde{D}_h$[~0.717]) (solid circles);
  otherwise, occurrence of LTD.

  (dotted horizontal line: representing J_0 (=0.2))

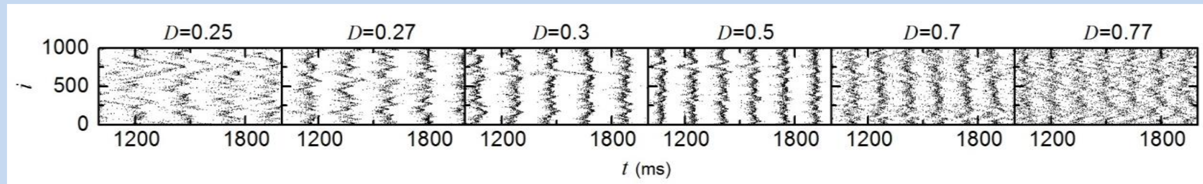


"Mathew" Effect of the STDP

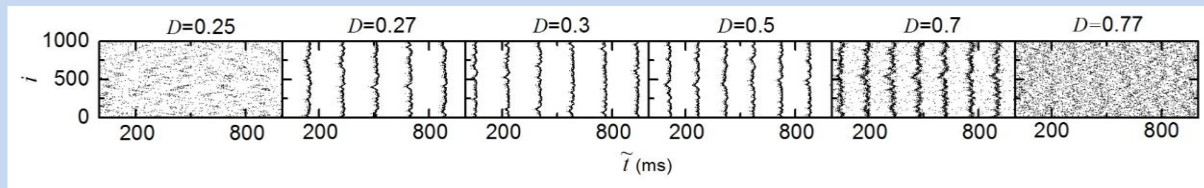
- **Effect of the STDP on the Synchronization Degree**

  LTP (LTD) $\rightarrow$ Increasing (decreasing) the degree of SSS

Absence of the STDP



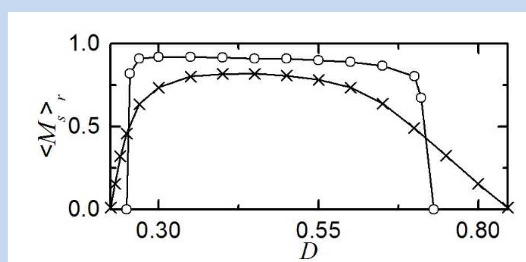
Presence of the STDP



- **Characterization of the Synchronization Degree via Statistical-Mechanical Spiking Measure M_s**

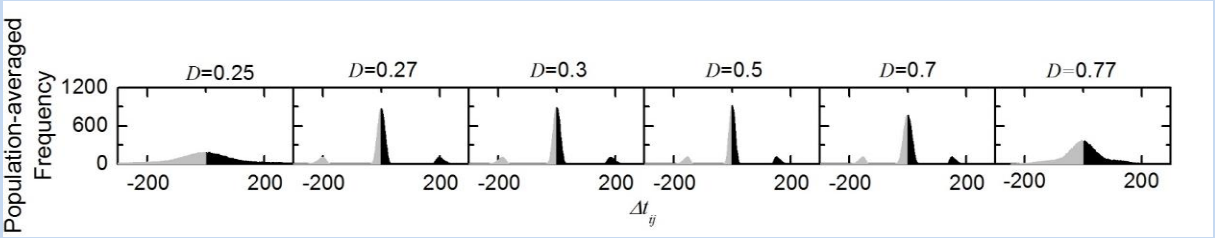
  Occurrence of "Mathew Effect" in Synaptic Plasticity:

  Good synchronization gets better via LTP, while bad synchronization gets worse via LTD.



Microscopic Investigation on Emergences of LTP and LTD

- **Population-Averaged Histogram $H(\Delta t_{ij})$ for the Distribution of $\{\Delta t_{ij}\}$**



Time Interval: From t=0 and the saturation time (t=2000 sec)

- LTP (D = 0.27, 0.3, 0.5, & 0.7): 3 peaks appear

  Main central peak: Pre- and post-synaptic spike times in the same stripe in the raster plot of spikes

  $\Delta t_{ij} > 0 \rightarrow$ LTP, $\Delta t_{ij} < 0 \rightarrow$ LTD

  Two minor left and right peaks: Pre- and post-synaptic spike times in the different nearest-neighboring stripes.

  Pre-synaptic stripe precedes post-synaptic stripe (causality) $\rightarrow$ right minor peak (LTP);

  Otherwise $\rightarrow$ left minor peak (LTD).

- LTD (D = 0.25 & 0.77): Population states: Desynchronized due to overlap of spiking stripes in the raster plot of spikes

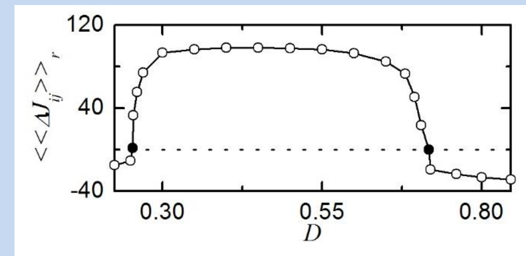
  $\rightarrow$ Merging of the main peak with the left and the right minor peaks due to overlap of spiking stripes in the raster plot of spikes

  $\Rightarrow$ Appearance of one broad main peak

Microscopic Investigation on Emergences of LTP and LTD (Continued)

- **Population-averaged synaptic modification $<<\Delta J_{ij}>>_r$**

$$\ll \Delta J_{ij} \gg_r \simeq \sum_{bins} H(\Delta t_{ij}) \cdot \Delta J_{ij}(\Delta t_{ij})$$

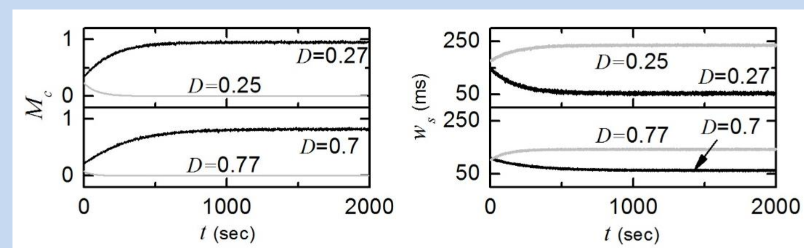


Population-averaged limit values of synaptic strengths $\ll J^*_{ij} \gg_r (= J_0 + \delta \ll \Delta J_{ij} \gg_r)$: Agree well with the directly-calculated values

- **Pair-Correlations between the Pre- and Post-Synaptic Instantaneous Individual Spike Rates (IISRs)**

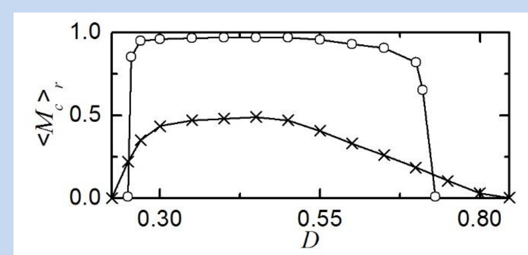
Microscopic correlation measure M_c:

Representing the average "in-phase" degree between the pre- and the post-synaptic pairs.



Strong M_c → Narrow width of stripes
→ Narrow distribution of $\{\Delta t_{ij}\}$ → LTP

Weak M_c → Wide width of stripes
→ Wide distribution of $\{\Delta t_{ij}\}$ → LTD



→ "Matthew" effect in M_c also occurs.

Effect of the Multiplicative STDP on the SSS

- **Multiplicative STDP**

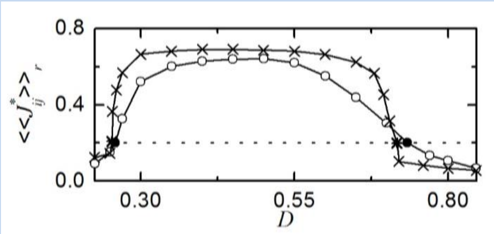
$$J_{ij} \rightarrow J_{ij} + (J^* - J_{ij}) | \delta \Delta J_{ij} (\Delta t_{ij}) |$$

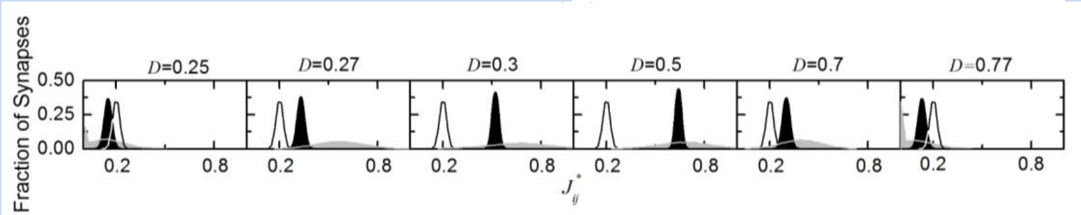
J^*=J_h (J_l) for LTP(LTD) [J_h=1 & J_l=0.0001]

- **Soft Transition for the Synaptic Strength J_{ij}**
  A gradual transition to LTP/LTD due to soft bounds

  Both $< J^*_{ij} >$ and its standard deviation are also smaller than those for the case of additive STDP.

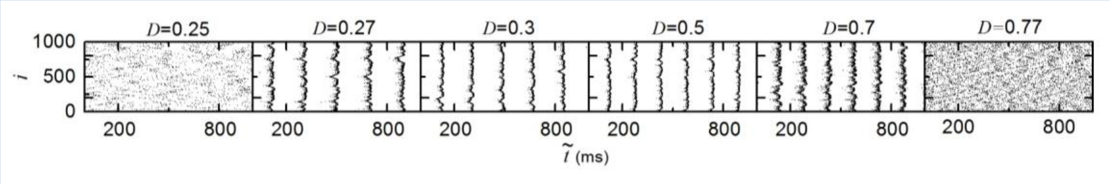
X: Additive STDP. O: Multiplicative STDP



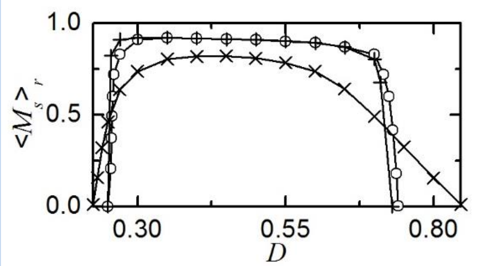


Black curve: Initial. Gray: Additive STDP. Black: Multiplicative STDP

- **Degree of SSS**
  Thanks to the smaller standard deviation, nearly the same as those in the additive case although $< J^*_{ij} >$ are smaller.



  Changes in M_s near the thresholds:
  Relatively less rapid due to soft bounds



Summary

- **Stochastic Spike Synchronization (SSS)**
  SSS between noise-induced spikings of subthreshold neurons occurs over a large range of intermediated noise intensities.

- **Effect of Spike-Timing-Dependent Plasticity of the SSS**
  "Matthew" effect in synaptic plasticity occurs.
  → Good synchronization gets better via long-term potentiation (LTP) of synaptic strengths, while bad synchronization gets worse via long-term depression (LTD).

- **Investigation of Emergences of LTP and LTD**
  Microscopic studies based on both the distributions of time delays between the pre- and the post-synaptic spike times and the pair-correlations between the pre- and the post-synaptic instantaneous individual spike rates.

- **Effect of Multiplicative STDP on the SSS**
  Occurrence of soft transition for the synaptic strength J_{ij}
  → Gradual transition to LTP/LTD due to the soft bounds in contrast to the hard bounds for the additive case.
  Changes in M_s near the thresholds are also relatively less rapid due to soft bounds, when compared with the additive case.