Stochastic Burst Synchronization in A Scale-Free Neural Network with Spike-Timing-Dependent Plasticity

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Synaptic Plasticity

- **Stochastic Burst Synchronization (SBS)**

  Subthreshold neurons: Fire only with the help of noise and exhibit irregular discharges like Geiger counter

  Bursting: Neuronal activity alternates, on a slow timescale, between a silent phase and an active (bursting) phase of fast repetitive spikings

  SBS: Population synchronization between complex noise-induced burstings of subthreshold neurons & correlated with brain function of encoding sensory stimuli in the noisy environment

  Previous works on SBS: Synaptic strengths were static.

- **Spike-Timing-Dependent Plasticity (STDP)**

  Synaptic Plasticity: In real brains synaptic strengths may vary to adapt to environment (potentiated or depressed)

  STDP: Plasticity depending on the relative time difference between the pre-and the post-synaptic burst onset times

- **Purpose of Our Study**

  Investigation of Effect of the STDP on the SBS in the Scale-Free Network (SFN)

Excitatory SFN of Subthreshold Izhikevich Neurons

- **Scale-Free Network (SFN) of Subthreshold Izhikevich Neurons**

  Barabási-Albert SFN with symmetric attachment degree l^*=10
  (Growth and preferential directed attachment with l_{in} incoming edges and l_{out} outgoing edges; $l_{in} = l_{out} = l^*$)

  Subthreshold Izhikevich Neurons for the DC current $I_{DC,i} \in [3.55, 3.65]$

- **Hebbian STDP**

  Update of coupling strengths: Additive nearest-burst pair-based STDP rule

$$J_{ij} \rightarrow J_{ij} + \delta \Delta J_{ij}(\Delta t_{ij})$$

$$\Delta t_{ij} = t_i^{(post)} - t_j^{(pre)}, \;\; \delta = 0.005$$

$$J_{ij} \in [J_l (= 0.0001), J_h (= 5.0)]$$

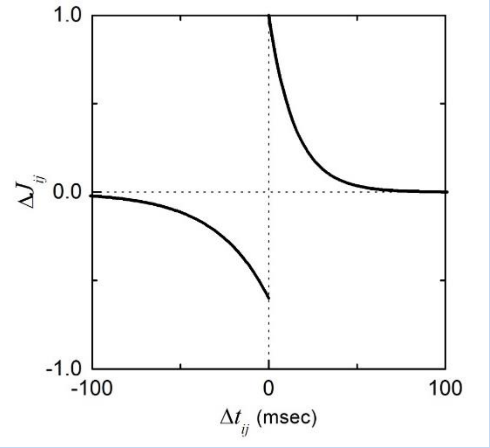
Initial synaptic strengths: Mean J_0=2.5 & standard deviation σ=0.02

Asymmetric time window for ΔJ_{ij}

$$\Delta J_{ij} = \begin{cases} A_+ e^{-\Delta t_{ij}/\tau_+} & \text{for } \Delta t_{ij} > 0 \\ -A_- e^{\Delta t_{ij}/\tau_-} & \text{for } \Delta t_{ij} < 0 \end{cases}$$

$$A_+ = 1.0, \; A_- = 0.6, \tau_+ = 15\text{msec}, \tau_- = 30\text{msec}, \Delta J_{ij}(\Delta t_{ij} = 0) = 0.$$

Δt_{ij} > 0 $\rightarrow$ LTP, Δt_{ij} < 0 $\rightarrow$ LTD



SBS in the Absence of STDP

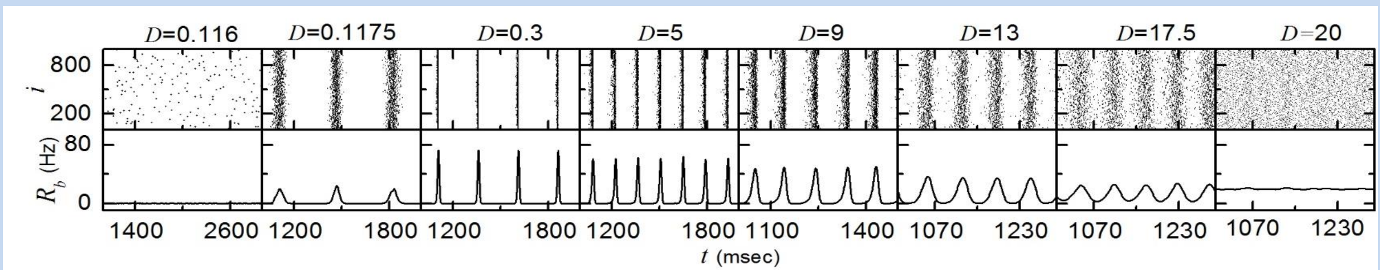
Initial coupling strengths $\{J_{ij}\}$: Gaussian distribution with mean J_0=2.5 and standard deviation σ_0=0.02

- **Raster Plots of Burst Onset Times**

Appearance of stripes in the raster plot for synchronous case

- **Instantaneous Population Burst Rate (IPBR)**

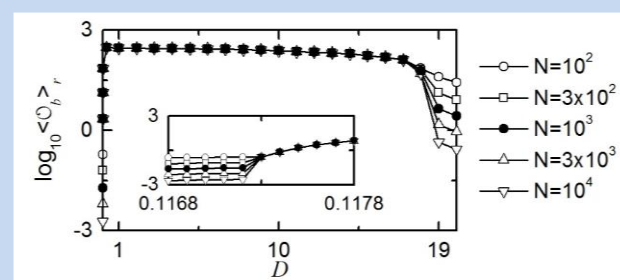
$$R_b(t) = \frac{1}{N} \sum_{i=1}^{N} \sum_{s=1}^{n_i} K_h(t - t_b^{(i)}); \qquad K_h(t) = \frac{1}{\sqrt{2\pi}h} e^{-t^2/2h^2}, -\infty < t < \infty$$



- **Thermodynamic Bursting Order Parameter:** $\mathcal{O}_b \equiv \overline{(R_b(t) - \overline{R_b(t)})^2}$

Synchronized (desynchronized) state:
$\mathcal{O}_b$ approach non-zero (zero) limit values for $N \to \infty$

SBS in $D_l^*(\simeq 0.1173) < D < D_h^*(\simeq 18.4)$ via competition between the constructive and the destructive roles of noise.

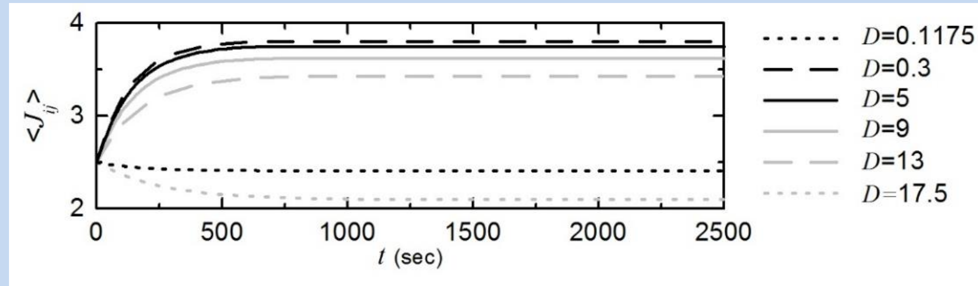


Effect of the STDP on the SBS

- **Time-Evolution of Population-Averaged Synaptic Strength** $<J_{ij}>$

D=0.3, 5, 9 and 13: $<J_{ij}>$ increases monotonically above its initial value J_0 (=2.5), and it approaches a saturated limit value $< J^*_{ij} >$ → LTP

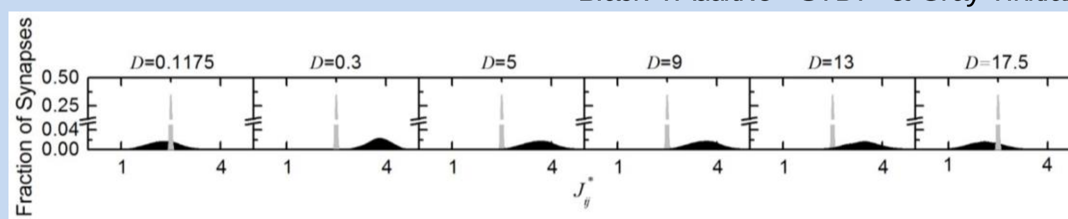
D=0.1175 and 17.5: $<J_{ij}>$ decreases monotonically below J_0, and it approaches $< J^*_{ij} >$ → LTD



- **Histograms for Fraction of Synapses** J^*_{ij} (Saturated limit value of J_{ij})

$< J^*_{ij} >$ becomes larger (smaller) than the initial value for the case of LTP (LTD). The standard deviations are very larger than the initial one (=0.02).

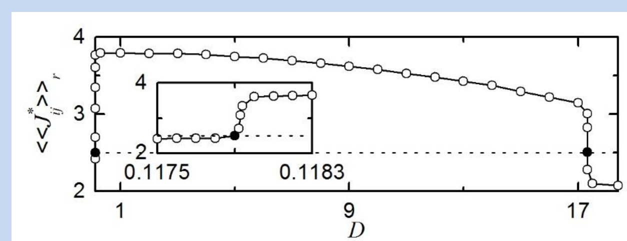
Black : Additive STDP & Gray : Initial



- **Population-Averaged Limit Values of Synaptic Strengths** $\langle\langle J^*_{ij} \rangle\rangle_r$

LTP occurs in ($\tilde{D}_l$[~0.1179], $\tilde{D}_h$[~17.336])

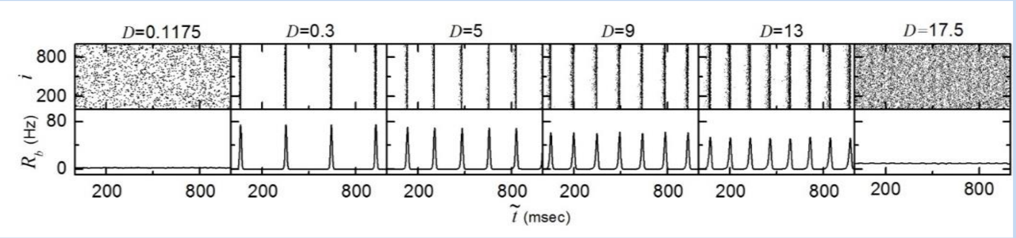
In most range of the SBS LTP occurs, while LTD takes place only near both ends.



"Mathew" Effect of the STDP

- **Raster Plots of Burst Onset Times IPBR $R_b(t)$**

LTP → The degrees of SBS are increased.
LTD → The population states become desynchronized.



- **Characterization of the Synchronization Degree via Statistical-Mechanical Bursting Measure M_b**

Pacing degree of the ith bursting stripe: averaging the contributions to $R_b(t)$ of all microscopic burst onset times in the ith bursting stripe

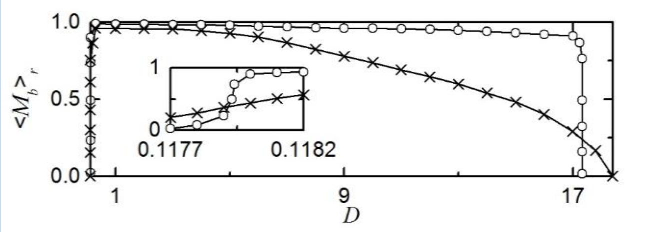
$$P_i^{(b)} = \frac{1}{B_i} \sum_{k=1}^{B_i} \cos \Phi_k^{(b)}$$

B_i: Number of burst onset times in the ith bursting stripe
$\Phi_k^{(b)}$: global phase of burst onset time

$$M_b = \frac{1}{N_b} \sum_{i=1}^{N_b} P_i^{(b)}$$

N_b: No. of bursting stripe

LTP → Good burst synchronization gets better.
LTD → Bad burst synchronization gets worse.



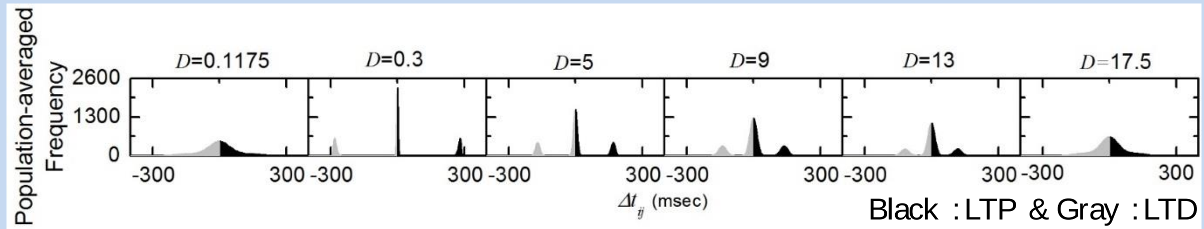
Open circles : Additive STDP & Crosses : Absence of STDP

Microscopic Investigation on Emergences of LTP and LTD

- **Population-Averaged Histograms $H(\Delta t_{ij})$ for $\{\Delta t_{ij}\}$ during t=0~saturation time t^* (=2000sec)**

LTP (D=0.3, 5, 9, & 13): 3 peaks.
One main central peaks (same bursting stripe) and two minor left and right peaks (different nearest-neighboring bursting stripes)

LTD (D=0.1175 & 17.5): Single broad peak via a merging of the above main and minor peaks

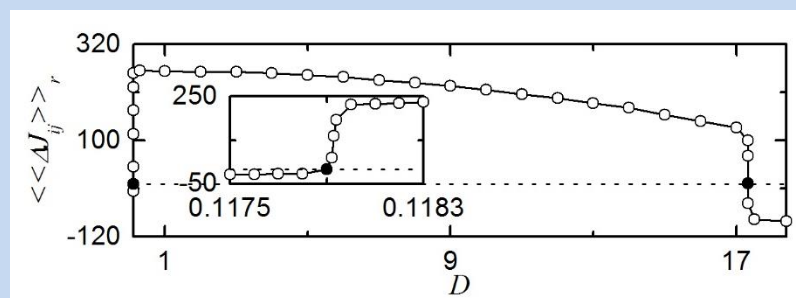


- **Population-Averaged Synaptic Modification $\langle\langle \Delta J_{ij} \rangle\rangle_r$ Obtained from $H(\Delta t_{ij})$**

Population-averaged limit values of synaptic strengths agree well with direct-obtained values.

$$\left\langle\left\langle \Delta J_{ij} \right\rangle\right\rangle_r \simeq \sum_{bins} H(\Delta t_{ij}) \cdot \Delta J_{ij}(\Delta t_{ij})$$

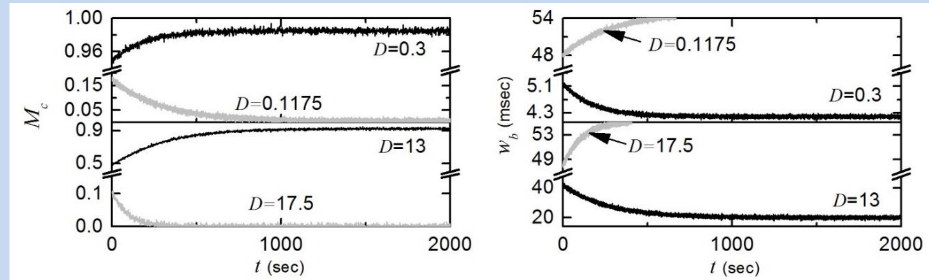
$$\langle\langle J_{ij}^* \rangle\rangle_r \; (= J_0 + \delta \langle\langle \Delta J_{ij} \rangle\rangle_r)$$



Microscopic Cross-Correlations between Synaptic Pairs

• Microscopic Correlation Measure M_c

M_c: Average "in-phase" degree between the pre- and the post-synaptic pairs



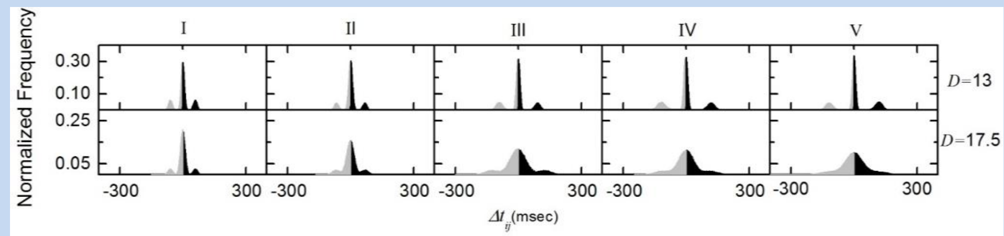
$$M_c = \frac{1}{N_{syn}} \sum_{(i,j)} C_{i,j}(0), \quad C_{i,j}(\tau) = \frac{\overline{\Delta r_i(t+\tau)\Delta r_j(t)}}{\sqrt{\overline{\Delta r_i^2(t)}}\sqrt{\overline{\Delta r_j^2(t)}}}$$

• Widths w_b of Bursting Stripes

Strong (weak) $M_c \rightarrow w_b$ decreases (increases) $\rightarrow$ Narrow (wide) distribution of $\Delta t_{ij} \rightarrow$ LTP (LTD)

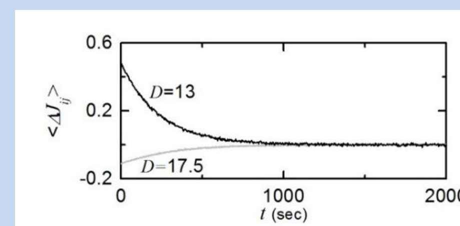
Black : LTP & Gray : LTD

• Time-Evolutions of Normalized Histogram $H(\Delta t_{ij})$ for $\{\Delta t_{ij}\}$



LTP: 3 peaks $\rightarrow$ Peaks become narrowed.
$\rightarrow$ Main peak becomes symmetric.
LTD: 3 peaks $\rightarrow$ Merged into the single broad peak $\rightarrow$ Peak becomes symmetric.

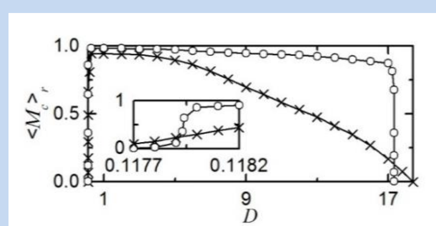
• Time-Evolutions of $<\Delta J_{ij}>$ Obtained from $H(\Delta t_{ij})$



D=13 (D=17.5): $<\Delta J_{ij}(t)>$ is positive (negative)
$<\Delta J_{ij}(t)>$ approach 0 because $H(\Delta t_{ij})$ become symmetric. $\rightarrow$ LTP (LTD)

• Mathew Effect in M_c

M_c : Matthew effect also occurs.



Open circles: Additive STDP
Crosses: Absence of STDP

Summary

- **Stochastic Burst Synchronization (SBS) in the Absence of STDP**
  - SBS between complex noise-induced burstings of subthreshold neurons:
    Correlated with brain function of encoding sensory stimuli
    in the noisy environment.
  - Occurrence of SBS in intermediated noise intensities via competition between
    the constructive and the destructive roles of noise.
  - Previous works on SBS: Synaptic strengths were static.

- **Investigation of The Effect of STDP on the SBS**
  - Occurrence of "Matthew" effect in synaptic plasticity
    → Good burst synchronization gets better via LTP, while bad burst
    synchronization gets worse via LTD.
  - Emergences of LTP and LTD: Intensively investigated via microscopic studies
    based on both the distributions of time delays between the pre-
    and the post-synaptic burst onset times and the pair-correlations
    between the pre- and the post-synaptic IIBRs.