Effect of Inhibitory Spike-Timing-Dependent Plasticity on Burst Synchronization in A Scale-Free Neuronal Network

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Introduction

- **Burst Synchronization (BS)**

Bursting: Neuronal activity alternates, on a slow timescale, between a silent phase and an active (bursting) phase of fast repetitive spikings [e.g., see the bursting behavior of HR neuron (given below)]

Representative bursting neurons: Bursting and chattering neurons in the cortex, thalamic relay neurons and thalamic reticular neurons in the thalamus, hippocampal pyramidal neurons, Purkinje cells in the cerebellum, pancreatic β-cells, and respiratory neurons in pre-Botzinger complex

BS: Population synchronization on the slow bursting timescale between the burst onset times. Associated with the fundamental brain function (e.g., learning, memory, and development) and neural diseases (e.g., Parkinson's disease and epilepsy)

Previous works on BS: Synaptic strengths were static [1].

- **Inhibitory Spike-Timing-Dependent Plasticity (iSTDP)**

Synaptic Plasticity: In real brains synaptic strengths may vary to adapt to environment (potentiated or depressed)

STDP: Plasticity depending on the relative time difference between the pre-and the post-synaptic burst onset times

Study of STDP: Mainly focused on excitatory synapses (eSTDP)

iSTDP: Less attention because of experimental obstacles and diversity of inhibitory interneurons. (With the advent of fluorescent labeling and optical manipulation iSTDP has begun to be focused.)

- **Purpose of Our Study**

Investigation of Effect of the iSTDP on the BS in the Scale-Free Network (SFN)

SFN of Inhibitory Hindmarsh-Rose (HR) Bursting Neurons

- **Governing Equations**

$$\frac{dx_i}{dt} = y_i - ax_i^3 + bx_i^2 - z_i + I_{DC,i} + D\xi_i - I_{syn,i}, \quad I_{syn,i} = \frac{1}{d_i^{in}} \sum_{j=1(j\neq i)}^{N} J_{ij} w_{ij} s_j(t)(x_i - X_{syn}),$$

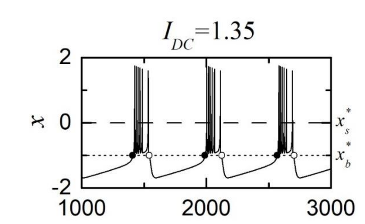
$$\frac{dy_i}{dt} = c - dx_i^2 - y_i, \quad s_j(t) = \sum_{f=1}^{F_j} E(t - t_f^{(j)} - \tau_l);$$

$$\frac{dz_i}{dt} = r[s(x_i - x_o) - z_i, \quad i = 1, \ldots, N, \quad E(t) = \frac{1}{\tau_d - \tau_r}(e^{-t/\tau_d} - e^{-t/\tau_r})\Theta(t).$$

$a=1, b=3, c=1, d=5, r=0.001, s=4, x_o=-1.6$

$\tau_l=1, \tau_r=0.5, \tau_d=5, X_{syn}=-2$

Suprathreshold Neurons: $I_{DC,i} \in [1.3, 1.4]$



Dotted lines: Bursting threshold
Dashed lines: Spiking threshold
Solid & open circles: Burst onset & offset times, respectively

- **Barabási-Albert SFN**
  - Growth and preferential directed attachment with l_{in} incoming edges and l_{out} outgoing edges
  - Power-law degree distribution → A few percent of hubs with exceptionally large number of connections
  - Symmetric attachment: $l_{in}=l_{out}=l^*$

BS for l^*=15 in the Absence of iSTDP

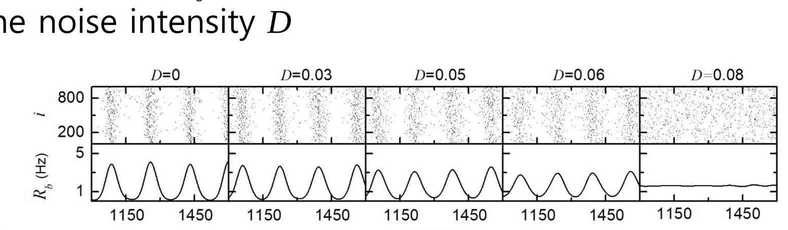
Initial coupling strengths $\{J_{ij}\}$: Gaussian distribution with mean J_0=12 and standard deviation σ=0.1

Aim: Investigation of emergence of BS by varying the noise intensity D

- **Raster Plots of Burst Onset Times**

Appearance of bursting stripes (composed of burst onset times and representing BS) for smaller values of D

Desynchronization for D=0.08 → Burst onset times: Completely scattered without forming any bursting stripes



- **Instantaneous Population Burst Rate (IPBR)**

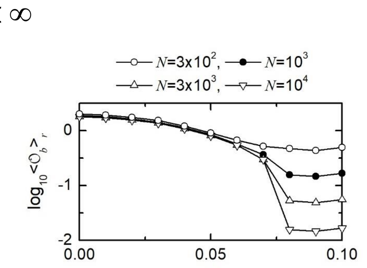
$$R_b(t) = \frac{1}{N}\sum_{i=1}^{N}\sum_{s=1}^{n_i} K_h(t - t_s^{(i)}); \quad K_h(t) = \frac{1}{\sqrt{2\pi}h} e^{-t^2/2h^2}, -\infty < t < \infty$$

- **Thermodynamic Bursting Order Parameter:**

Synchronized (desynchronized) state: $\mathcal{O}_b \equiv \overline{(R_b(t) - \overline{R_b(t)})^2}$

$\mathcal{O}_b$ approach non-zero (zero) limit values for $N\to\infty$

Occurrence of sparse BS for $D<D^*$ (≃ 0.072)



BS for l^*=15 in the Absence of iSTDP (continued)

- **Statistical-Mechanical Bursting Measure M_b**

Statistical-mechanical bursting measure M_b: Given by the product of the occupation and the pacing degrees of burst onset times in the raster plot

$$M_i^{(b)} = O_i^{(b)} \cdot P_i^{(b)}$$

Occupation degree $<O_i^{(b)}>$: representing the average density of stripes in the raster plot

$$O_i^{(b)} = \frac{N_i^{(b)}}{N}$$ $N_i^{(b)}$: No. of bursting neurons in the ith bursting stripe

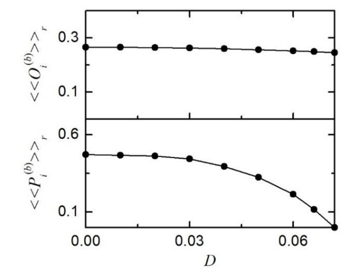
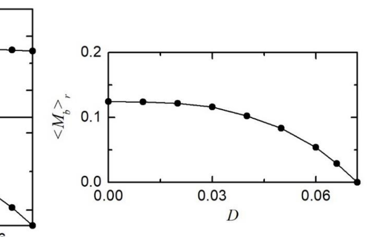
Pacing degree $<P_i^{(b)}>$: representing the average smearing of stripes in the raster plot (average contribution of all microscopic burst onset times to the IPBR)

$$P_i^{(b)} = \frac{1}{B_i}\sum_{k=1}^{B_i} \cos\Phi_k^{(b)}$$

B_i: Number of burst onset times in the ith bursting stripe
$\Phi^{(b)}_k$: global phase of burst onset time
N_b: No. of bursting stripe

$$M_b = \frac{1}{N_b}\sum_{i=1}^{N_b} P_i^{(b)}$$

With increasing D from 0 to D^*, $<<O_i^{(b)}>>_r$ is nearly constant and $<<P_i^{(b)}>>_r$ & $<M_b>_r$ decrease to zero due to complete overlap of sparse bursting stripes.

Anti-Hebbian STDP

- **Multiplicative STDP Rule:** $J_{ij} \to J_{ij} + \delta(J^* - J_{ij})|\Delta J_{ij}(\Delta t_{ij})|$;

δ (=0.08): Update rate. $J_{ij} \in [J_l, J_h]$; $J_l = 0.0001$ & $J_h = 20$

$J^* = J_h (J_l)$ for LTP (LTD).

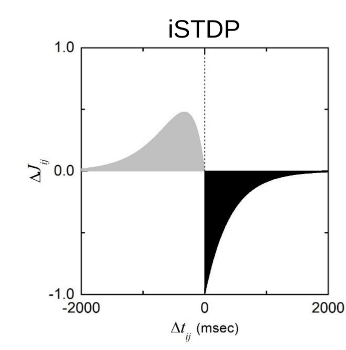
$\Delta t_{ij} (= t_i^{(post)} - t_j^{(pre)})$: Time difference between the nearest burst onset times of the post-synaptic neuron i and the pre-synaptic neuron j.

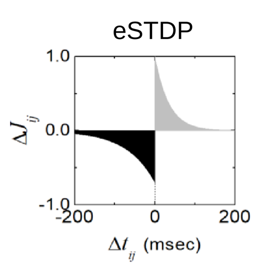
- **Time Window for the Anti-Hebbian STDP**

$$\Delta J_{ij} = \begin{cases} -A_+ e^{-\Delta t_{ij}/\tau_+} & \text{for } \Delta t_{ij} > 0 \\ -A_- \frac{\Delta t_{ij}}{\tau_-} e^{\Delta t_{ij}/\tau_-} & \text{for } \Delta t_{ij} \le 0 \end{cases}$$

$A_+ = 1.0, A_- = 1.3,$
$\tau_+ = 410$ msec, $\tau_- = 330$ msec

When a post synaptic burst follows a pre-synaptic burst (Δt_{ij}>0), LTD appears; otherwise (Δt_{ij}≤0), LTP occurs (c.f. eSTDP: Hebbian STDP [2]; Δt_{ij}>0 → LTP, Δt_{ij}<0 → LTD)



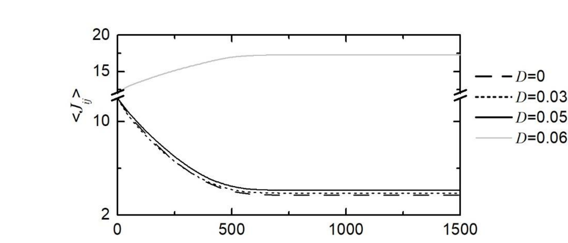


Effect of the iSTDP on BS for l^*=15

- **Time-Evolutions of Population-Averaged Synaptic Strength $<J_{ij}>$**

D=0, 0.03, 0.05 and 13: $<J_{ij}>$ decreases monotonically below its initial value J_0 (=12), and it approaches a saturated limit value $<J_{ij}^*>$ → LTD

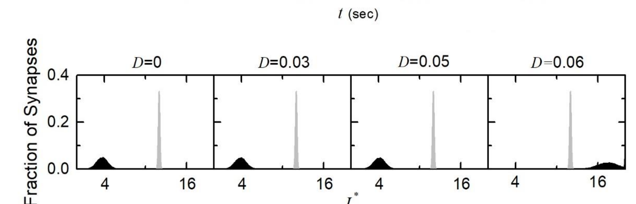
D=0.06: $<J_{ij}>$ increases monotonically above J_0, and it approaches $<J_{ij}^*>$ → LTP



- **Histograms for Fraction of Synapses J_{ij}^***

$<J_{ij}^*>$ becomes smaller (larger) than the initial value for the case of LTD (LTP).

The standard deviations are very larger than the initial one (=0.1).

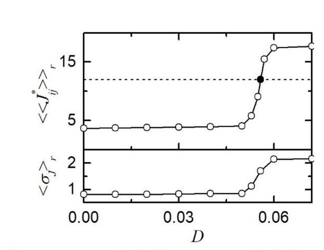


- **Population-Averaged Limit Values of Synaptic Strengths $<<J_{ij}^*>>_r$ versus D**

LTD occurs in $0 \le D < D_{th}$ (D_{th}~0.0558).

LTP takes place in a smaller range of BS (D_{th}<D<D^*).

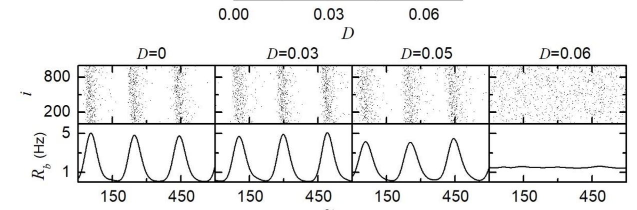
$<\sigma_J>_r$ are larger than the initial value for both cases of LTD & LTP.



- **Raster Plots of Burst Onset Times and IPBR $R_b(t)$**

LTD → The degrees of BS are increased.

LTP → The population states become desynchronized.

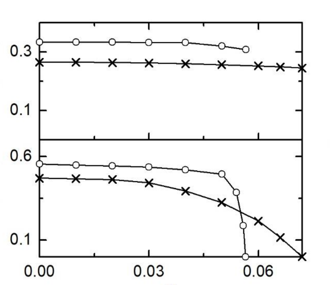
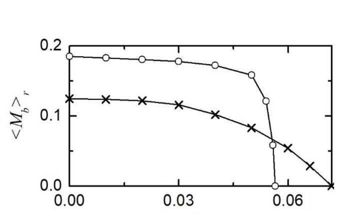


- **"Matthew" Effect in Inhibitory Synaptic Plasticity**

Occupation degree: Increase due to LTD (decreased mean synaptic inhibition)

Pacing degree: Increase due to dominant effect of LTD (overcoming the effect of increased standard deviation σ_J).

M_b: Similar to those of pacing degree (occupation degree are nearly constant)

Open circles : iSTDP & Crosses : Absece of iSTDP

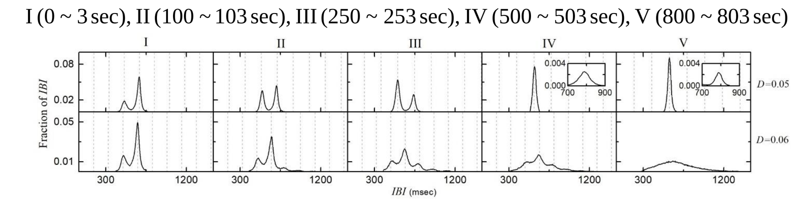
→ Good BS gets better via LTD, while bad BS gets worse via LTP.

Microscopic Investigation on Emergences of LTD and LTP for l^*=15

- **Time-Evolutions of The IBI Histograms**

LTD (D=0.05):
Stage I: Two peaks at $3T_G$ and $4T_G$
With increasing the stage, the amplitude of the 1st peak increases, while that of the 2nd peak is reduced.

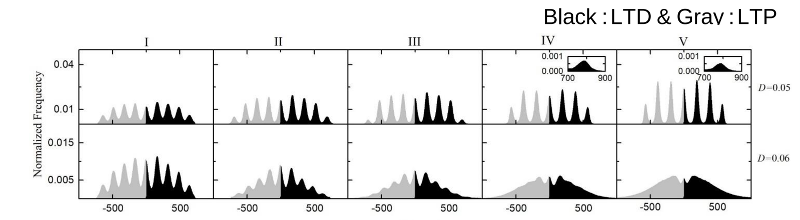
LTP (D=0.06): With increasing the stage, peaks become wider, and merging between them occurs.
Stage V: Single broad peak



- **Time-Evolution Normalized Histograms $H(\Delta t_{ij})$ for the Distribution of $\{\Delta t_{ij}\}$**

LTP (D=0.05): Multi-peaks appear
Stage I: Effect of right black part (causality) is dominant. → LTD
As t is increased: Peaks become narrow and sharper. → Increasing the degree of BS
Effect of LTD (black part) tends to cancel out the effect of LTP (gray part)

LTD (D=0.06):
Stage I: Effect of left gray part (acausality) is dominant. → LTP
As t is increased: peaks become wider and merging. → Decreasing the degree of BS
→ Appearance of one broad single peak



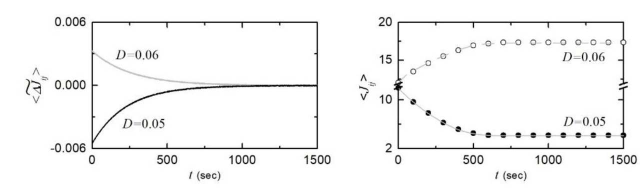
- **Population-Averaged Multiplicative Synaptic Modification $<\Delta\tilde{J}_{ij}(\Delta t_{ij})>_k$ Obtained from $H(\Delta t_{ij})$**

Population-averaged synaptic strength: $<J_{ij}>_k = <J_{ij}>_{k-1} + \delta \cdot <\Delta\tilde{J}_{ij}(\Delta t_{ij})>_k$

Population-averaged multiplicative synaptic modification:

$$<\Delta\tilde{J}_{ij}(\Delta t_{ij})>_k \simeq (J^* - <J_{ij}>_{k-1}) <|\Delta J_{ij}(\Delta t_{ij})|>_k,$$

$$<|\Delta J_{ij}(\Delta t_{ij})|>_k \simeq \sum_{bins} H_k(\Delta t_{ij}) \cdot |\Delta J_{ij}(\Delta t_{ij})|$$



Population-averaged limit values of synaptic strengths agree well with direct-obtained values.

Summary

- **Burst Synchronization (BS) in the Absence of iSTDP**
  - BS: Population synchronization on the slow bursting timescale between the burst onset times associated with the fundamental brain function and neural diseases
  - Previous works on BS: Synaptic strengths were static.
- **Inhibitory STDP (iSTDP)**
  - Study of STDP: Mainly focused on excitatory synapses (eSTDP)
  - iSTDP: Less attention because of experimental obstacles and diversity of inhibitory interneurons.
  - Anti-Hebbian time window in contrast to the Hebbian time window for eSTDP
- **Investigation of The Effect of iSTDP on the BS**
  - Occurrence of "Matthew" effect in synaptic plasticity → Good burst synchronization gets better via LTD, while bad burst synchronization gets worse via LTP.
  - Due to inhibition, the roles of LTD (increasing the degree of BS) and LTP (decreasing the degree of BS) for the case of iSTDP are reversed in comparison with those in eSTDP where the degree of population synchronization is increased (decreased) via LTP (LTD).
  - Emergences of LTD and LTP: Intensively investigated via microscopic studies based on the distributions of time delays between the pre- and the post-synaptic burst onset times

References

[1] S.-Y. Kim and W. Lim, "Effect of network architecture on burst and spike synchronization in a scale-free network of bursting neurons," Neural Netw. 79, 53-77 (2016).
[2] S.-Y. Kim and W. Lim, "Effect of spike-timing-dependent plasticity on stochastic burst synchronization in a scale-free neuronal network," Cogn. Neurodyn. 12, 315-342 (2018)
[3] S.-Y. Kim and W. Lim, "Burst synchronization in a scale-free neuronal network with inhibitory spike-timing-dependent plasticity," Accepted in Cogn. Neurodyn., bioRxiv: DOI:10.1101/321562 (2018).

Acknowledgments

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