Effect of Spike-Timing-Dependent Plasticity on Stochastic Burst Synchronization in A Scale-Free Neural Network

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Introduction

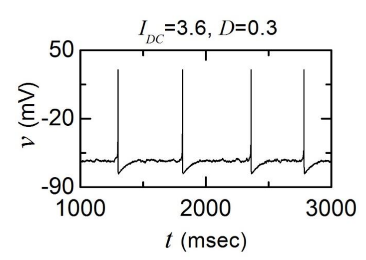
- **Stochastic Burst Synchronization (SBS)**
  Subthreshold neurons: Fire only with the help of noise and exhibit irregular discharges like Geiger counter
  Bursting: Neuronal activity alternates, on a slow timescale, between a silent phase and an active (bursting) phase of fast repetitive spikings
  SBS: Population synchronization between complex noise-induced burstings of subthreshold neurons & correlated with brain function of encoding sensory stimuli in the noisy environment
  Previous works on SBS: Synaptic strengths were static.
- **Spike-Timing-Dependent Plasticity (STDP)**
  Synaptic Plasticity: In real brains synaptic strengths may vary to adapt to environment (potentiated or depressed)
  STDP: Plasticity depending on the relative time difference between the pre-and the post-synaptic burst onset times
- **Purpose of Our Study**
  Investigation of Effect of the STDP on the SBS in the Scale-Free Network (SFN)

Excitatory SFN of Subthreshold Izhikevich Regular Spiking Neurons

- **Governing Equations**

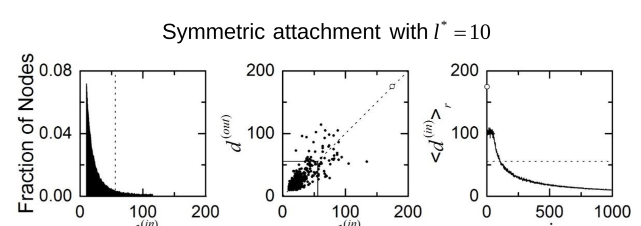
$$\frac{dv_i}{dt} = f(v_i) - u_i + I_{DC,i} + D\xi_i - I_{syn,i},$$
$$\frac{du_i}{dt} = a(bv_i - u_i), \quad i = 1, \dots, N,$$
if $v_i \ge v_p$, then $v_i \leftarrow c$ and $u_i \leftarrow u_i + d$.
$a = 0.02, b = 0.2, c = -65, d = 8, v_p = 30$
$\tau_l = 1, \tau_r = 0.5, \tau_d = 2, V_{syn} = 0$

$$f(v) = 0.04v^2 + 5v + 140,$$
$$I_{syn,i} = \frac{1}{d_i^{in}} \sum_{j=1(j\ne i)}^{N} J_{ij} w_{ij} s_j(t)(v_i - V_{syn}),$$
$$s_j(t) = \sum_{f=1}^{F_j} E(t - t_f^{(j)} - \tau_l);$$
$$E(t) = \frac{1}{\tau_d - \tau_r}(e^{-t/\tau_d} - e^{-t/\tau_r})\Theta(t).$$

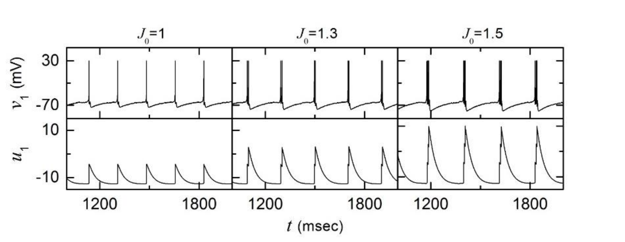


Subthreshold Neurons: $I_{DC,i} \in$ [3.55, 3.65]

- **Barabási-Albert SFN**
  - Growth and preferential directed attachment with l_{in} incoming edges and l_{out} outgoing edges
  - Power-law degree distribution
  - Hub group (outside rectangle): Head hub (with highest degree) & secondary hub (with higher degrees)
  - Peripheral group (inside rectangle): Majority of nodes (with lower degrees)
  - Symmetric attachment: l_{in}=l_{out}=l^*
    Asymmetric attachment: l_{in}=l^*+Δl, l_{out}=l^*+Δl



- **Coupling-Induced Bursting**
  When J passes a threshold for a fixed value of D=0.3, noise-induce burstings occur.

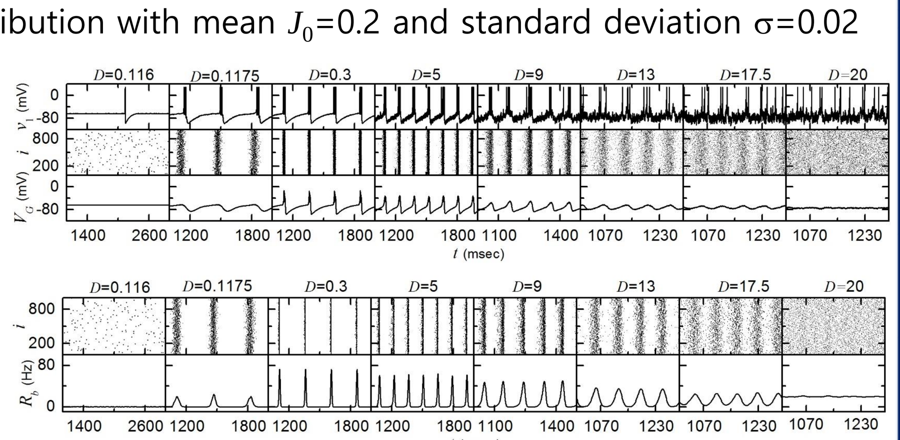


SBS for l^*=10 in the Absence of STDP

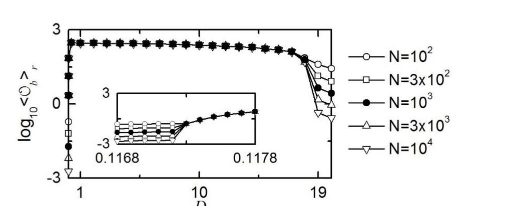
Initial coupling strengths {J_{ij}}: Gaussian distribution with mean J_0=0.2 and standard deviation σ=0.02

- **Raster Plots of Spikes and Burst Onset Times**
  Appearance of stripes in the raster plot for synchronous case
- **Instantaneous Population Burst Rate (IPBR)**

$$R_b(t) = \frac{1}{N}\sum_{i=1}^{N}\sum_{s=1}^{n_i} K_h(t - t_b^{(i)});$$
$$K_h(t) = \frac{1}{\sqrt{2\pi}h} e^{-t^2/2h^2}, -\infty < t < \infty$$



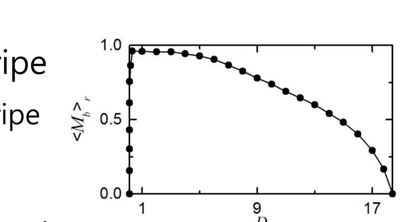
- **Thermodynamic Bursting Order Parameter:** $\mathcal{O}_b \equiv \overline{(R_b(t) - \overline{R_b(t)})^2}$
  Synchronized (desynchronized) state:
  $\mathcal{O}_b$ approach non-zero (zero) limit values for $N \to \infty$
  SBS in D^*_l (≃ 0.1173)<D<D^*_h (≃ 18.4) via competition between the constructive and the destructive roles of noise.



- **Statistical-Mechanical Bursting Measure M_b**
  Pacing degree of the ith bursting stripe: averaging the contributions to $R_b(t)$ of all microscopic burst onset times in the ith bursting stripe

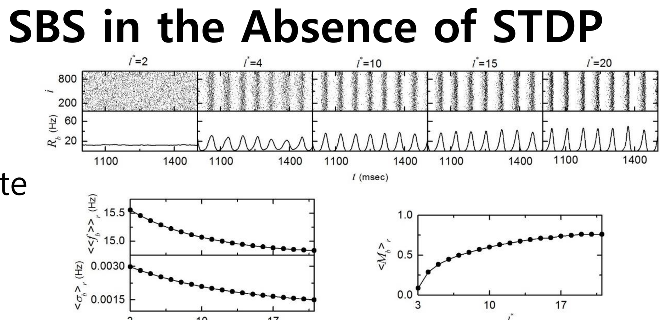
$$P_i^{(b)} = \frac{1}{B_i}\sum_{k=1}^{B_i}\cos\Phi_k^{(b)}$$
B_i: Number of burst onset times in the ith bursting stripe
$\Phi^{(b)}_k$: global phase of burst onset time
N_b: No. of bursting stripe

$$M_b = \frac{1}{N_b}\sum_{i=1}^{N_b} P_i^{(b)}$$
Rapid increase passing D_l^*, slow decrease, and relative rapid decrease for large D

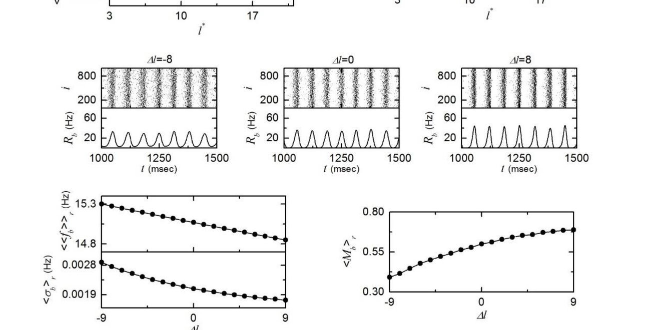


Effect of Scale-Free Connectivity on the SBS in the Absence of STDP

- **Symmetric Attachment**
  Increasing l^*: Better efficiency of global communication
  & Decrease in population-average mean bursting rate $\langle f_b \rangle$ and standard deviation σ_b
  → Increase in the degree of SBS

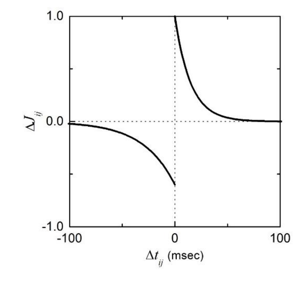


- **Asymmetric Attachment**
  Increasing (decreasing) Δl from 0: Worse efficiency of global communication
  & Decrease (increasing) in $\langle f_b \rangle$ and σ_b
  → Increase (decrease) in the degree of SBS
  Better individual dynamics overcome worse efficiency of communication



Hebbian STDP

- **Additive STDP Rule:** $J_{ij} \to J_{ij} + \delta\,\Delta J_{ij}(\Delta t_{ij});\ \delta = 0.005,\ J_{ij} \in [0.0001, 5.0]$
  $\Delta t_{ij} (= t_i^{(post)} - t_j^{(pre)})$: Time difference between the nearest burst onset times of the post-synaptic neuron i and the pre-synaptic neuron j.

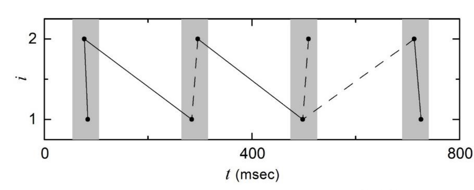


- **Time Window for the Hebbian STDP**

$$\Delta J_{ij} = \begin{cases} A_+ e^{-\Delta t_{ij}/\tau_+} & \text{for } \Delta t_{ij} > 0 \\ -A_- e^{\Delta t_{ij}/\tau_-} & \text{for } \Delta t_{ij} < 0 \end{cases}$$
$A_+ = 1.0,\ A_- = 0.6,\ \tau_+ = 15$ msec, $\tau_- = 30$ msec, $\Delta J_{ij}(\Delta t_{ij} = 0) = 0.$

When a post synaptic burst follows a pre-synaptic burst (Δt_{ij}>0), LTP appears; otherwise (Δt_{ij}<0), LTD occurs

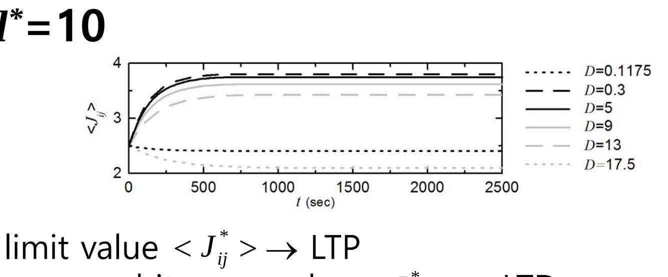
- **Nearest-Burst Pair-Based STDP Rule**
  LTP/LTD may occur between the pre- and the post-synaptic burst onset times in the same or the nearest-neighboring bursting stripes.



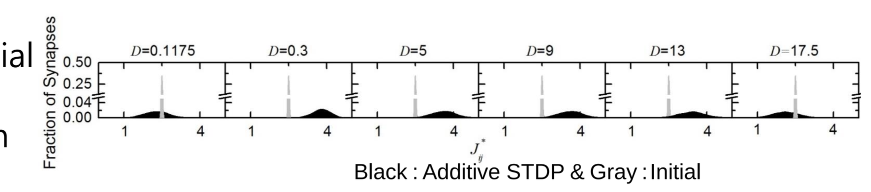
Solid line : LTP
Dashed line : LTD

Effect of the Additive STDP on SBS for l^*=10

- **Time-Evolutions of Population-Averaged Synaptic Strength** $\langle J_{ij} \rangle$
  D=0.3, 5, 9 and 13: $\langle J_{ij} \rangle$ increases monotonically above its initial value J_0 (=0.2), and it approaches a saturated limit value $\langle J^*_{ij} \rangle$ → LTP
  D=0.1175 and 17.5: $\langle J_{ij} \rangle$ decreases monotonically below J_0, and it approaches $\langle J^*_{ij} \rangle$ → LTD

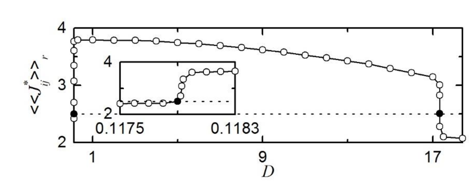


- **Histograms for Fraction of Synapses** J^*_{ij}
  $\langle J^*_{ij} \rangle$ becomes larger (smaller) than the initial value for the case of LTP (LTD).
  The standard deviations are very larger than the initial one (=0.02).

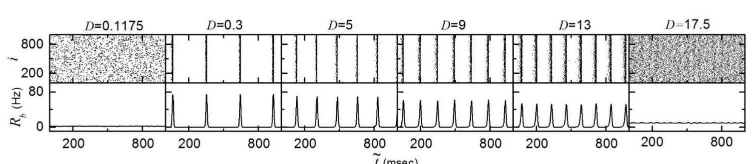


Black : Additive STDP & Gray : Initial

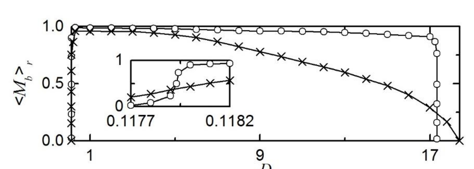
- **Population-Averaged Limit Values of Synaptic Strengths** $\langle\langle J^*_{ij} \rangle\rangle_r$ **versus** D
  LTP occurs in ($\widetilde{D}_l$, $\widetilde{D}_h$); ($\widetilde{D}_l$≃0.1179, $\widetilde{D}_h$≃17.336).
  In most range of the SBS LTP occurs, while LTD takes place only near both ends.



- **Raster Plots of Burst Onset Times and IPBR $R_b(t)$**
  LTP → The degrees of SBS are increased.
  LTD → The population states become desynchronized.



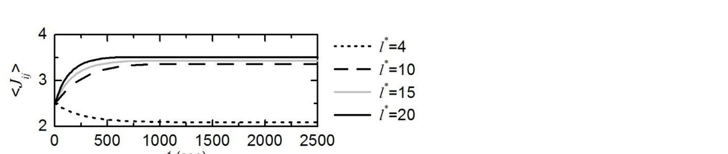
- **"Matthew" Effect in M_b**
  LTP → Good burst synchronization gets better.
  LTD → Bad burst synchronization gets worse.

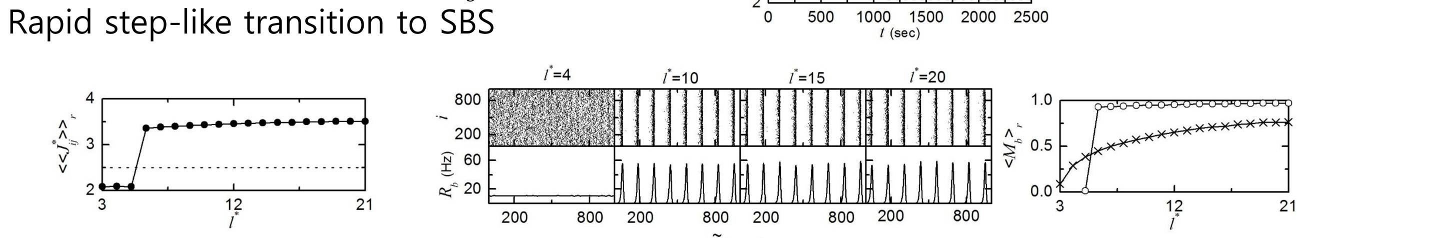


Open circles : Additive STDP & Crosses : Absece of STDP

Effect of Network Architecture on SBS in the Presence of STDP

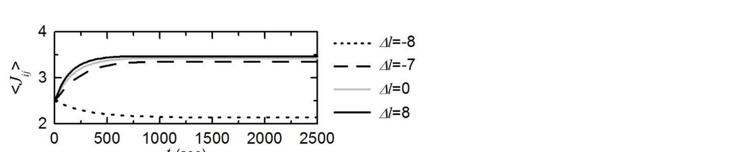
- **Symmetric Attachment**
  Occurrence of LTP (LTD) for l^*≥6 (l^*≤5)
  Occurrence of Matthew Effect in M_b:
  Rapid step-like transition to SBS

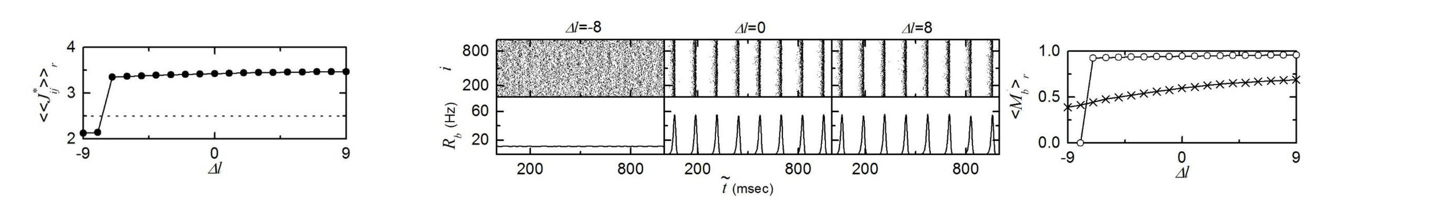




Open circles : Additive STDP & Crosses : Absece of STDP

- **Asymmetric Attachment**
  Occurrence of LTP (LTD) for Δl≥-7 (Δl≤-8)
  Occurrence of Matthew Effect in M_b



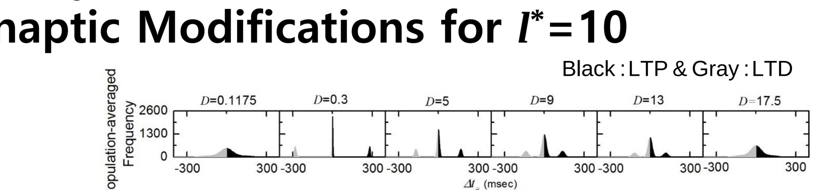


Open circles : Additive STDP & Crosses : Absece of STDP

Distribution of Microscopic Time Delays between the Pre- and the Post-Synaptic Burst Onset Times and Synaptic Modifications for l^*=10

- **Population-Averaged Histograms $H(\Delta t_{ij})$ for {Δt_{ij}} during t=0~saturation time t^*** (=2000sec)
  LTP (D=0.3, 5, 9, & 13): 3 peaks.
  One main central peaks (same bursting stripe) and two minor left and right peaks (different nearest-neighboring bursting stripes)
  LTD (D=0.1175 & 17.5): Single broad peak via a merging of the above main and minor peaks

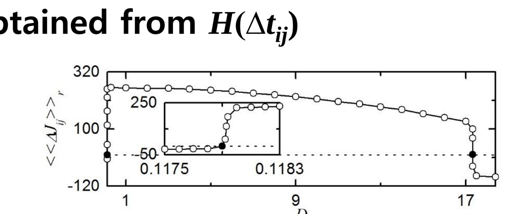
Black : LTP & Gray : LTD



- **Population-Averaged Synaptic Modification $\langle\langle \Delta J_{ij} \rangle\rangle_r$ Obtained from $H(\Delta t_{ij})$**

$$\langle\langle \Delta J_{ij} \rangle\rangle_r \simeq \sum_{bins} H(\Delta t_{ij}) \cdot \Delta J_{ij}(\Delta t_{ij})$$

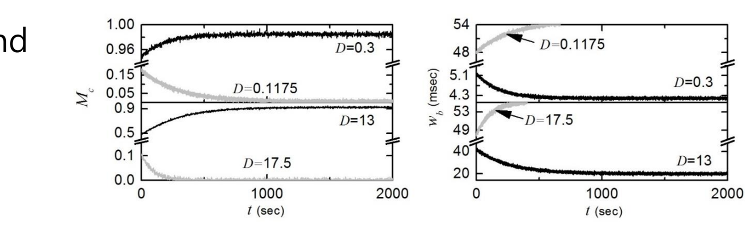
Population-averaged limit values of synaptic strengths $\langle\langle J^*_{ij} \rangle\rangle_r (= J_0 + \delta \langle\langle \Delta J_{ij} \rangle\rangle_r)$ agree well with direct-obtained values.



Microscopic Cross-Correlations between Synaptic Pairs for l^*=10

- **Microscopic Correlation Measure M_c**
  M_c: Average "in-phase" degree between the pre- and the post-synaptic pairs

$$M_c = \frac{1}{N_{syn}}\sum_{(i,j)} C_{i,j}(0), \quad C_{i,j}(\tau) = \frac{\overline{\Delta r_i(t+\tau)\Delta r_j(t)}}{\sqrt{\overline{\Delta r_i^2(t)}}\sqrt{\overline{\Delta r_j^2(t)}}}$$

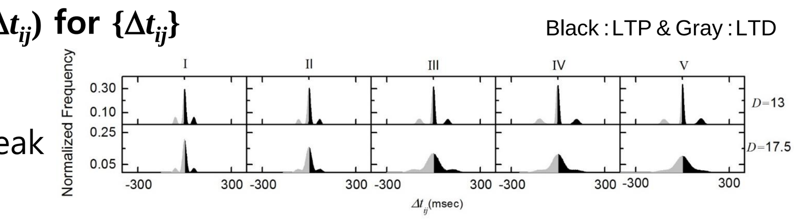


D=0.3 & 13 (D=0.27 & 0.7): M_c increases and approaches a limit value (zero).

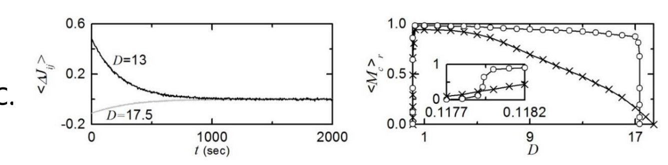
- **Widths w_b of Bursting Stripes**
  Strong (weak) M_c → w_b decreases (increases) → Narrow (wide) distribution of Δt_{ij} → LTP (LTD)
- **Time-Evolutions of Normalized Histogram $H(\Delta t_{ij})$ for {Δt_{ij}}**

Black : LTP & Gray : LTD

  LTP: 3 peaks → Peaks become narrowed.
  → Main peak becomes symmetric.
  LTD: 3 peaks → Merged into the single broad peak
  → Peak becomes symmetric.



- **Time-Evolutions of $\langle \Delta J_{ij} \rangle$ Obtained from $H(\Delta t_{ij})$**
  D=13 (D=17.5): $\langle \Delta J_{ij}(t) \rangle$ is positive (negative) approach 0 because $H(\Delta t_{ij})$ become symmetric.
  → LTP (LTD)



- **Mathew Effect in M_c** M_c : Matthew effect also occurs.

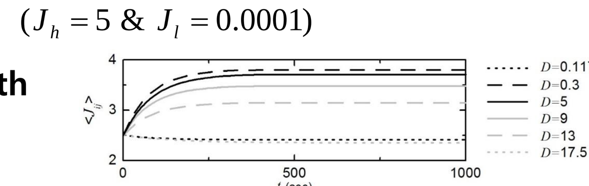
Open circles : Additive STDP & Crosses : Absece of STDP

Effect of the Multiplicative STDP on the SBS for l^*=10

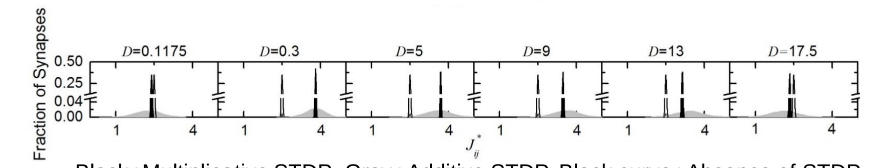
- **Multiplicative STDP Rule**: Soft bound for the synaptic strength

$$J_{ij} \to J_{ij} + (J^* - J_{ij})|\,\delta \Delta J_{ij}(\Delta t_{ij})| \qquad J^* = J_h(J_l) \text{ for the LTP (LTD)} \quad (J_h = 5 \ \& \ J_l = 0.0001)$$

- **Time-Evolutions of Population-Averaged Synaptic Strength**
  Soft bound → Saturation time is shorter and $\langle J^*_{ij} \rangle$ are smaller than the additive STDP case.



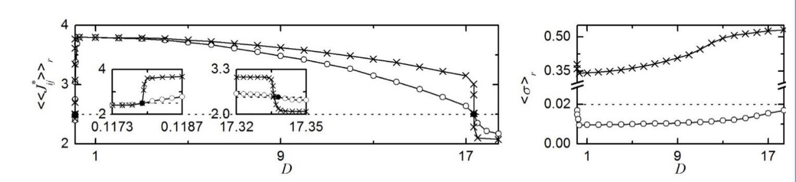
- **Histograms for Fraction of** J^*_{ij}
  D=0.3, 5, 9, & 13 (LTP): $\langle J^*_{ij} \rangle$ becomes larger than the initial value.
  D=0.1175 & 17.5 (LTD): $\langle J^*_{ij} \rangle$ becomes smaller than the initial value



Black : Multiplicative STDP, Gray : Additive STDP, Black curve : Absence of STDP

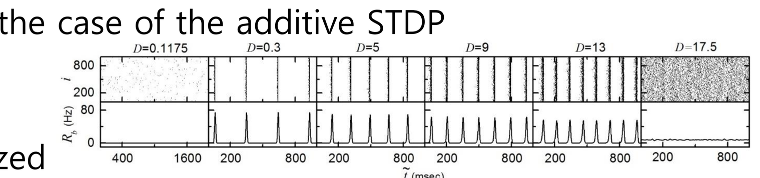
Both cases, standard deviations σ of {J^*_{ij}} are smaller than those for the case of additive STDP and the initial value (=0.02).

- **Population-Averaged Limit Values of Synaptic Strengths** $\langle\langle J^*_{ij} \rangle\rangle_r$ **versus** D
  LTP occurs in ($\widetilde{D}^*_l$, $\widetilde{D}^*_h$); ($\widetilde{D}^*_l$≃0.1179, $\widetilde{D}^*_h$≃17.338).
  Both $\langle J^*_{ij} \rangle$ and its standard deviation σ are smaller than those for the case of the additive STDP.
  Gradual change near the thresholds in contrast to the case of the additive STDP

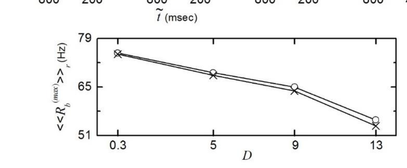


Open circles : Multiplicative STDP, Crosses : Additive STDP

- **Raster Plots of Burst Onset Times and IPBR $R_b(t)$**
  LTP → The degrees of SBS are increased.
  LTD → The population states become desynchronized



Effect of smaller σ (increasing the degree of SBS):
Much more than the effect of smaller $\langle J^*_{ij} \rangle$ (decreasing the degree of SBS) → Average maximum $\langle R_b^{(max)} \rangle$ of $R_b(t)$:
A little larger for the multiplicative case.



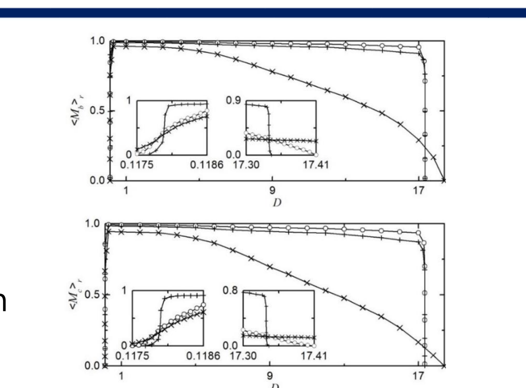
Open circles : Multiplicative STDP & Crosses : Additive STDP

Effect of Multiplicative STDP on M_b and M_c

Occurrence of Matthew effect in both M_b and M_c

Effect of smaller σ (increasing M_b and M_c): Much more than the effect of smaller $\langle J^*_{ij} \rangle$ (decreasing M_b and M_c)
→ M_b and M_c: A little larger than those in the additive case

Changes in M_b and M_c near the thresholds: Relatively less rapid than the additive case due to soft bounds



Open circles : Multiplicative STDP, Pluses : Additive STDP, Crosses : Absence of STDP

Summary

- **Stochastic Burst Synchronization (SBS) in the Absence of STDP**
  - SBS between complex noise-induced burstings of subthreshold neurons: Correlated with brain function of encoding sensory stimuli in the noisy environment.
  - Occurrence of SBS in intermediated noise intensities via competition between the constructive and the destructive roles of noise.
  - Previous works on SBS: Synaptic strengths were static.
- **Investigation of The Effect of Additive STDP on the SBS**
  - Occurrence of "Matthew" effect in synaptic plasticity
    → Good burst synchronization gets better via LTP, while bad burst synchronization gets worse via LTD.
  - Effect of the network architecture on SBS: Investigated by varying the degree of symmetric attachment and asymmetric attachment parameter
  - Emergences of LTP and LTD: Intensively investigated via microscopic studies based on both the distributions of time delays between the pre- and the post-synaptic burst onset times and the pair-correlations between the pre- and the post-synaptic IIBRs.
- **Investigation of The Effect of Multiplicative STDP on the SBS**
  - Soft bounds (a change in synaptic strengths scales linearly with the distance to the higher and the bounds) in the synaptic strengths in contrast to the hard bounds for the additive case.
  - Occurrence of Matthew effect in M_b.
  - Relatively less rapid transition near the thresholds, in comparison to the rapid transition for the additive case.
  - Both $\langle J^*_{ij} \rangle$ and the standard deviation σ of {J^*_{ij}}: Smaller than those for the case of additive STDP
  - Degrees of SBS in most LTP cases: A little larger than those in the additive case.