Effect of Interpopulation Spike-Timing-Dependent Plasticity on Neuronal Synchronized Rhythms in Clustered Small-World Networks with Inhibitory and Excitatory Populations

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Introduction

- **Fast Sparsely Synchronization (FSS)**
  - Population level: Fast synchronous oscillations [e.g. gamma rhythm (30~100 Hz) during awake behaving states and rapid eye movement sleep]
  - Cellular level: Stochastic and intermittent spike discharges at much lower rates than the population oscillation frequency
  - Related to diverse cognitive functions (e.g. multisensory feature binding, selective attention, and memory formation)
- **Synaptic Plasticity**
  - Adaptation of synapses in real brain: Synaptic strengths may vary to adapt to environment (potentiated or depressed)
  - Associated with brain functions (learning, memory, and development) and neural diseases (Parkinson's disease and epilepsy)
- **Spike-Timing-Dependent Plasticity (STDP)**
  - STDP: Plasticity depending on the relative time difference between the pre- and the post-synaptic spike times
  - Study of synaptic plasticity: Mainly focused on excitatory-to-excitatory (E to E) synapses
  - Synaptic plasticity at inhibitory synapse: Less attention due to experimental obstacles and diversity of inhibitory interneurons.
    (With the advent of fluorescent labeling and optical manipulation inhibitory synaptic plasticity has begun to be focused.)
    Particularly studies on inhibitory STDP at inhibitory-to-excitatory (I to E) synapses
- **Purpose of Our Study**
  Investigation of Effect of Interpopulation (I to E and E to I) STDPs on FSS in Clustered SWNs with Two Inhibitory and Excitatory Populations.

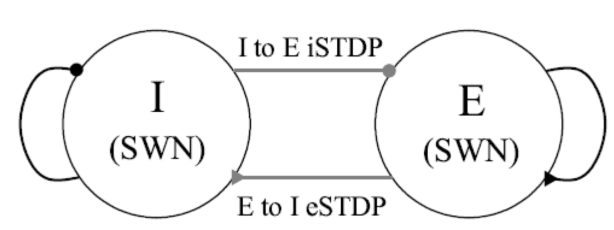
Clustered Small-World Networks Composed of Both I- and E-Populations

- **Governing Equations**

$$
\begin{aligned}
C_I \frac{dv_i^{(I)}}{dt} &= k_I(v_i^{(I)} - v_r^{(I)})(v_i^{(I)} - v_t^{(I)}) - u_i^{(I)} + \overline{I_i}^{(I)} + D_I \xi_i^{(I)} - I_{syn,i}^{(II)} - I_{syn,i}^{(IE)}, \\
\frac{du_i^{(I)}}{dt} &= a_I\{U(v_i^{(I)}) - u_i^{(I)}\}, \quad i = 1, \cdots, N_I, \\
C_E \frac{dv_i^{(E)}}{dt} &= k_E(v_i^{(E)} - v_r^{(E)})(v_i^{(E)} - v_t^{(E)}) - u_i^{(E)} + \overline{I_i}^{(E)} + D_E \xi_i^{(E)} - I_{syn,i}^{(EE)} - I_{syn,i}^{(EI)}, \\
\frac{du_i^{(E)}}{dt} &= a_E\{U(v_i^{(E)}) - u_i^{(E)}\}, \quad i = 1, \cdots, N_E,
\end{aligned}
$$

$$\text{if } v_i^{(X)} \geq v_p^{(X)}, \text{ then } v_i^{(X)} \leftarrow c_X \text{ and } u_i^{(X)} \leftarrow u_i^{(X)} + d_X, \quad (X = I \text{ or } E)$$

$$
\begin{aligned}
U(v^{(I)}) &= \begin{cases} 0 \text{ for } v^{(I)} < v_b^{(I)} \\ b_I(v^{(I)} - v_b^{(I)})^3 \text{ for } v^{(I)} \geq v_b^{(I)} \end{cases}, \\
U(v^{(E)}) &= b_E(v^{(E)} - v_b^{(E)}), \\
I_{syn,i}^{(XX)}(t) &= \frac{1}{d_{in,i}^{intra}} \sum_{j=1(j\neq i)}^{N_X} J_{ij}^{(XX)} w_{ij}^{(XX)} s_j^{(XX)}(t)(v_i^{(X)} - V_{syn}^{(X)}), \\
I_{syn,i}^{(XY)}(t) &= \frac{1}{d_{in,i}^{inter}} \sum_{j=1}^{N_Y} J_{ij}^{(XY)} w_{ij}^{(XY)} s_j^{(XY)}(t)(v_i^{(X)} - V_{syn}^{(Y)}), \\
s_j^{(XY)}(t) &= \sum_{f=1}^{F_j} E_{XY}(t - t_f^{(j)} - \tau_l^{(XY)}) \; (X = Y \text{or} X \neq Y); \\
E_{XY}(t) &= \frac{1}{\tau_d^{(XY)} - \tau_r^{(XY)}} (e^{-t/\tau_d^{(XY)}} - e^{-t/\tau_r^{(XY)}}) \Theta(t).
\end{aligned}
$$



Suprathreshold case: $\overline{I_i}^{(I)} = \overline{I_i}^{(E)} = \overline{I_i}$; $\overline{I_i} \in [680, 720]$

- **Clustered Small-World Networks**
  - Watts-Strogatz Small-world network consisting of N_I (N_E) (N_E: N_I = 4: 1) FS interneurons (RS pyramidal cells)
  - Random connections between two I- & E- small-world networks

$N_I = 600$ $\quad M_{syn}^{(I)} = 40$ $\quad N_E = 2400$ $\quad M_{syn}^{(E)} = 160$ $\quad p_{wiring} = 0.25$ $\quad p_{inter} = 1/15$

Interpopulation (I to E and E to I) STDP

- **Update of synaptic strength: Nearest-spike pair-based STDP rule**

$$J_{ij}^{(XY)} \rightarrow J_{ij}^{(XY)} + \delta(J^{(XY)^*} - J_{ij}^{(XY)}) |\Delta J_{ij}^{(XY)}(\Delta t_{ij}^{(XY)})|$$

$$\Delta t_{ij}^{(XY)} = t_i^{(post,X)} - t_j^{(pre,Y)}, J_{ij}^{(XY)} \in [J_l(= 0.0001), J_h(= 2000)]$$

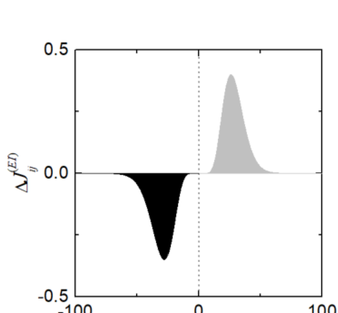
Initial interpopulation synaptic strengths: Gaussian distribution with mean $J_0^{(EI)}$=800, $J_0^{(IE)}$=487.5 & standard deviation σ_0 = 5

- **Delayed Hebbian I to E iSTDP**

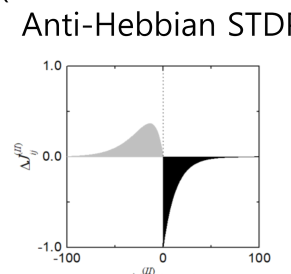
$$\Delta J_{ij}^{(EI)} = \begin{cases} E_+(t) \Delta t_{ij}^{(EI)^\beta} \text{ for } \Delta t_{ij}^{(EI)} > 0 \\ E_-(t) \Delta t_{ij}^{(EI)^\beta} \text{ for } \Delta t_{ij}^{(EI)} < 0 \end{cases}$$

$$E_+(t) = A_+ N_+ e^{-\Delta t_{ij}/\tau_+}; \; E_-(t) = -A_- N_- e^{\Delta t_{ij}/\tau_-}$$

$$N_+ = \frac{e^\beta}{\beta^\beta \cdot \tau_+^\beta} \qquad N_- = \frac{e^\beta}{\beta^\beta \cdot \tau_-^\beta}$$



(cf. I to I iSTDP: Anti-Hebbian STDP)

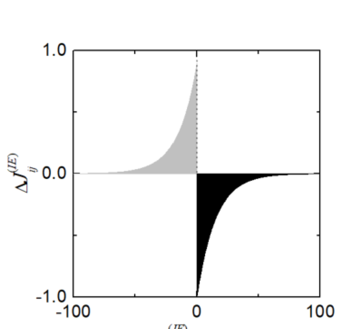


$\Delta t_{ij}^{(EI)} > 0 \rightarrow$ iLTP, $\Delta t_{ij}^{(EI)} < 0 \rightarrow$ iLTD

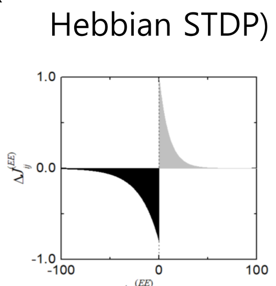
$A_+ = 0.4$, $A_- = 0.35$, $\tau_+ = 2.6$, $\tau_- = 2.8$, $\beta = 10$, $\delta = 0.1$

- **Anti-Hebbian E to i eSTDP**

$$\Delta J_{ij}^{(IE)} = \begin{cases} -A_+ \exp(-\Delta t_{ij}^{(IE)}/\tau_+) \;\; \text{for } \Delta t_{ij}^{(IE)} > 0 \\ A_- \exp(\Delta t_{ij}^{(IE)}/\tau_-) \;\; \text{for } \Delta t_{ij}^{(IE)} < 0 \end{cases}$$



(cf. E to E eSTDP: Hebbian STDP)

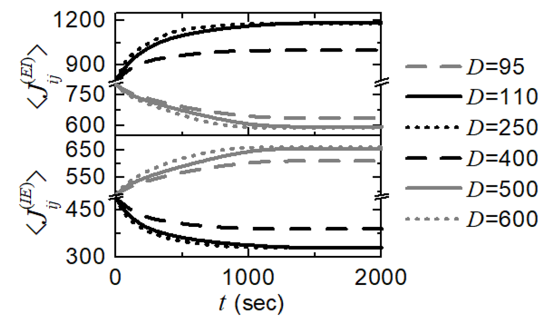


$\Delta t_{ij}^{(IE)} > 0 \rightarrow$ eLTD, $\Delta t_{ij}^{(IE)} < 0 \rightarrow$ eLTP

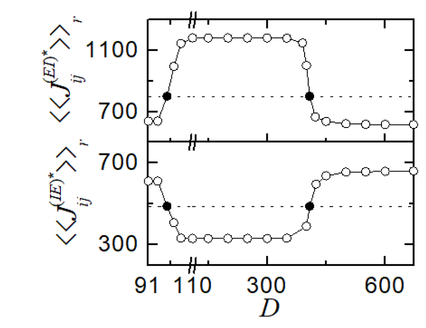
$A_+ = 1.0$, $A_- = 0.9$, $\tau_+ = 15.0$, $\tau_- = 15.0$, $\delta = 0.05$

Long-term Potentiation (LTP) and Depression (LTD)

- **FSS in the Absence of STDP**
  Occurrence of FSS in the range of $D_1^*(\simeq 91) < D < D_2^*(\simeq 537)$
- **Time-Evolution of Population-Averaged Synaptic Strength $<J_{ij}^{(EI)}>$ & $<J_{ij}^{(IE)}>$**
  D=110, 250, 400 (intermediate D): Monotonic increase (decrease) in $<J_{ij}^{(EI)}>$ ($<J_{ij}^{(IE)}>$) above $J_0^{(EI)}$ (below $J_0^{(IE)}$) and saturated to limit value → iLTP (eLTD)
  D=95, 500, 600: (small & large D) Monotonic decrease (increase) in $<J_{ij}^{(EI)}>$ ($<J_{ij}^{(IE)}>$) below $J_0^{(EI)}$ (above $J_0^{(IE)}$) and saturated to limit value → iLTD (eLTP)



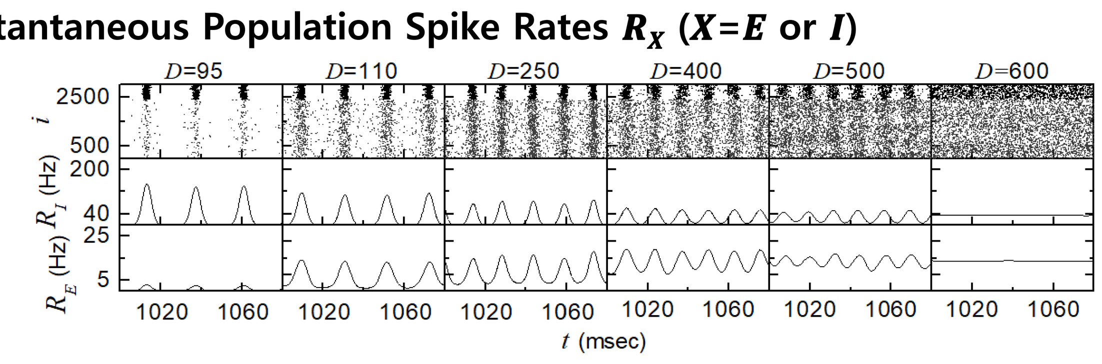
- **Population-Averaged Saturated Limit Values of Synaptic Strengths $<<J_{ij}^{(EI)*}>>_r$ & $<<J_{ij}^{(IE)*}>>_r$**
  Occurrence of iLTP & eLTD in an intermediate region $[\widetilde{D}_l(\simeq 99) < D < \widetilde{D}_h(\simeq 408)]$:
  $<<J_{ij}^{(EI)*}>>_r$: Increase & $<<J_{ij}^{(IE)*}>>_r$: Decrease
  Otherwise, occurrence of iLTD & eLTP in the regions of small & large D :
  $<<J_{ij}^{(EI)*}>>_r$: Decrease & $<<J_{ij}^{(IE)*}>>_r$: Increase
  $<<J_{ij}^{(EI)*}>>_r$: Bell-shaped graph. $<<J_{ij}^{(IE)*}>>_r$: Well-shaped graph.



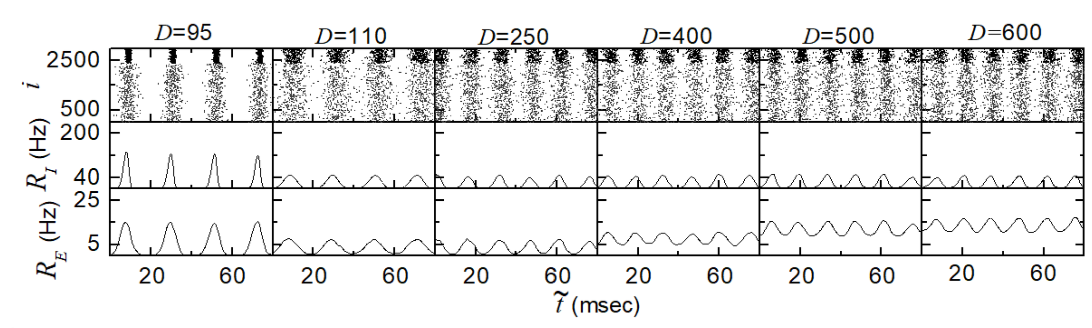
Effect of the Interpopulation STDPs on the FSS

- **Raster Plots of Spikes and Instantaneous Population Spike Rates R_X (X=E or I)**

Absence of STDP



Presence of interpopulation STDPs

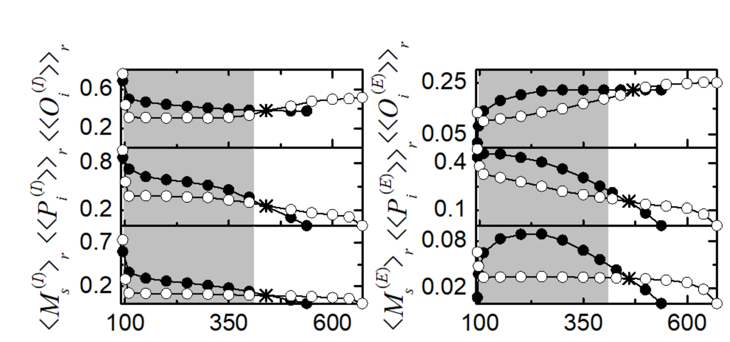


Black: I-population, Gray: E-population

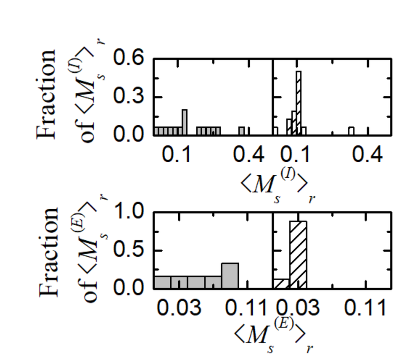
D=110, 250, 400 (intermediate D)
Decrease in degree of FSS (Decrease in amplitudes of R_X)
Due to increased I to E synaptic inhibition (iLTP) and decreased E to I synaptic excitation (eLTD)
D=95, 500, 600: (small & large D)
Increase in degree of FSS (Increase in amplitudes of R_X)
Due to decreased I to E synaptic inhibition (iLTD) and increased E to I synaptic excitation (eLTP)

Equalization Effect in Interpopulation Synaptic Plasticity

- **Characterization of Population Behaviors for FSS**
  FSS → Successive appearance of sparse spiking stripes in the raster plot of spikes
  Average occupation degree $<O_i^{(X)}>$: Density of spikes in the spiking stripes
  Average pacing degree $<P_i^{(X)}>$: Degree of phase coherence between spikes
  Spiking measure $M_s^{(X)}$: Product of $<O_i^{(X)}>$ & $<P_i^{(X)}>$
  Intermediate D region (iLTP & eLTD: Gray region):
  Decrease in $<O_i^{(X)}>$, $<P_i^{(X)}>$, & $M_s^{(X)}$
  Large & Small D regions (iLTD & eLTP):
  Increase in $<O_i^{(X)}>$, $<P_i^{(X)}>$, & $M_s^{(X)}$



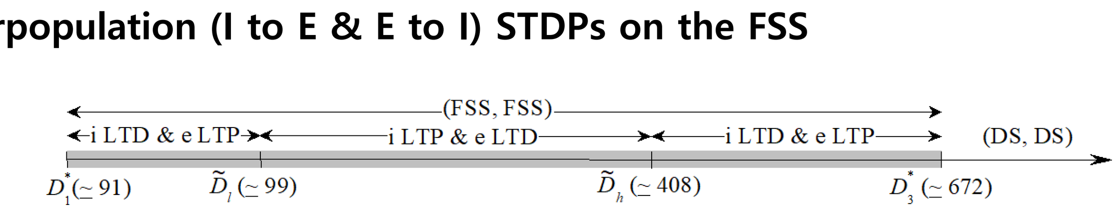
  $<O_i^{(X)}>$: Relatively fast-increasing function
  → Non-equalization effect with larger standard deviation
  $<P_i^{(X)}>$: Slowly-decreasing function → Weak equalization effect with smaller standard deviation
  → $M_s^{(X)}$: Flat in a wide region of intermediate and large D (strong equalization effect)



- **Strong Equalization Effect in $M_s^{(X)}$**
  Cooperative interplay between the weak equalization effect in decreasing $<P_i^{(X)}>$ and the non-equalization effect in increasing $<O_i^{(X)}>$
  → Strong equalization effect in $M_s^{(X)}$ with much smaller standard deviation

Summary

- **Fast Sparsely Synchronization (FSS) in the Absence of STDP**
  - FSS (related to diverse cognitive functions) occurs in the clustered small-world networks with two inhibitory and excitatory populations.
- **Effect of Interpopulation (I to E & E to I) STDPs on the FSS**



  - Degree of good synchronization gets decreased, while degree of bad synchronization becomes increased.
  - Degree of FSS becomes nearly the same in a wide range of noise intensity.
    → Occurrence of Strong Equalization Effect (also, occurrence of dumbing- down effect)

cf. Matthew effect in Intrapopulation (I to I & E to E) synaptic plasticity:
Good (bad) synchronization becomes better (worse).

[1] S.-Y. Kim & W. Lim, Neural Netw. 106, 50 (2018). [2] S.-Y. Kim & W. Lim, Neural Netw. 97, 92 (2018).