# Equalization Effect in Interpopulation Spike-Timing-Dependent Plasticity in Neuronal Networks with Inhibitory and Excitatory Populations

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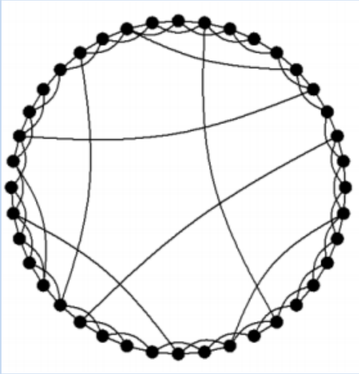
- **Fast Sparsely Synchronization**
  - Population level: Fast synchronous oscillations [e.g. gamma rhythm (30~100 Hz) during awake behaving states and rapid eye movement sleep]
  - Cellular level: Stochastic and intermittent spike discharges at much lower rates than the population oscillation frequency
  - Related to diverse cognitive functions (e.g. multisensory feature binding, selective attention, and memory formation)

- **Small-World Network (SWN)**
  - Architecture of synaptic connections in real brain: Complex topology neither regular nor completely random
  - Small-World Network: Predominantly local connections and rare long-range connections
    $\rightarrow$ High local clustering and short average path length



# Interpopulation STDPs

- **Synaptic Plasticity**
  - Adaptation of synapses in real brain: Synaptic strengths may vary to adapt to environment (potentiated or depressed)
  - Associated with brain functions (learning, memory, and development) and neural diseases (Parkinson's disease and epilepsy)

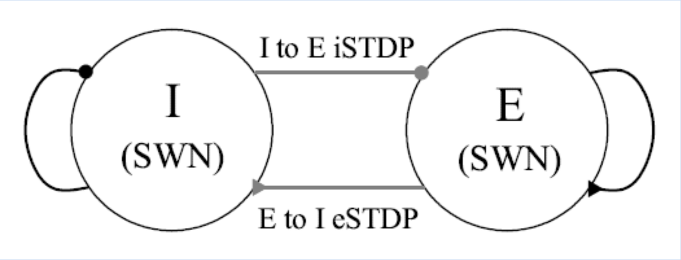
- **Spike-Timing-Dependent Plasticity (STDP)**
  - STDP: Plasticity depending on the relative time difference between the pre-and the post-synaptic spike times
  - Study of synaptic plasticity: Mainly focused on excitatory-to-excitatory (E to E) synapses
  - Synaptic plasticity at inhibitory synapse: Less attention due to experimental obstacles and diversity of inhibitory interneurons.
    (With the advent of fluorescent labeling and optical manipulation inhibitory synaptic plasticity has begun to be focused.)
    Particularly studies on inhibitory STDP at inhibitory-to-excitatory (I to E) synapses

- **Purpose of Our Study**

  Investigation of Effect of Interpopulation (I to E and E to I) STDPs on Fast Sparsely Synchronization in Clustered Small-World Networks with Two Inhibitory and Excitatory Populations.

# Clustered SWNs Composed of Two I- & E-Populations

- **Clustered SWNs**
  - Watts-Strogatz SWN consisting of $N_I$ ($N_E$) ($N_E : N_I = 4:1$) FS interneurons (RS pyramidal cells)
  - Random connections between two I-SWN & E-SWN




- **Interpopulation (I to E and E to I) STDP**
  - Update of synaptic strength: Nearest-spike pair-based STDP rule:

$$J_{ij}^{(XY)} \rightarrow J_{ij}^{(XY)} + \delta (J^{(XY)^*} - J_{ij}^{(XY)}) | \Delta J_{ij}^{(XY)} (\Delta t_{ij}^{(XY)}) | \qquad \Delta t_{ij}^{(XY)} = t_i^{(post,X)} - t_j^{(pre,Y)}, J_{ij}^{(XY)} \in [J_l (= 0.0001), J_h (= 2000)]$$

  - Initial interpopulation synaptic strengths: Gaussian distribution with mean $J_0^{(EI)}$=800, $J_0^{(IE)}$=487.5 & standard deviation $\sigma_0 = 5$

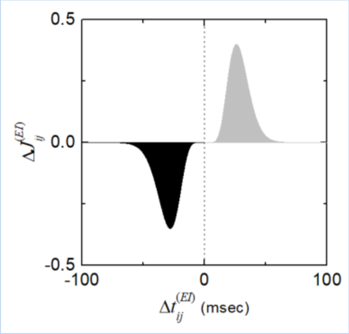
**- Delayed Hebbian I to E iSTDP**

$$\Delta J_{ij}^{(EI)} = \begin{cases} E_+(t) \Delta t_{ij}^{(EI)\beta} & \text{for } \Delta t_{ij}^{(EI)} > 0 \\ E_-(t) \Delta t_{ij}^{(EI)\beta} & \text{for } \Delta t_{ij}^{(EI)} < 0 \end{cases} \quad E_+(t) = A_+ N_+ e^{-\Delta t_{ij} / \tau_+}; \; E_-(t) = -A_- N_- e^{\Delta t_{ij} / \tau_-}$$

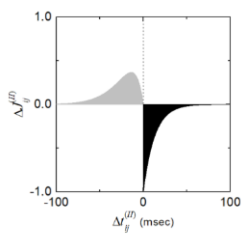
$$N_+ = \frac{e^\beta}{\beta^\beta \cdot \tau_+^\beta} \quad N_- = \frac{e^\beta}{\beta^\beta \cdot \tau_-^\beta}$$

$\Delta t_{ij}^{(EI)} > 0 \rightarrow$ iLTP, $\Delta t_{ij}^{(EI)} < 0 \rightarrow$ iLTD

$A_+ = 0.4$
$A_- = 0.35$
$\tau_+ = 2.6$
$\tau_- = 2.8$
$\beta = 10$
$\delta = 0.1$




(cf. I to I iSTDP: Anti-Hebbian STDP)

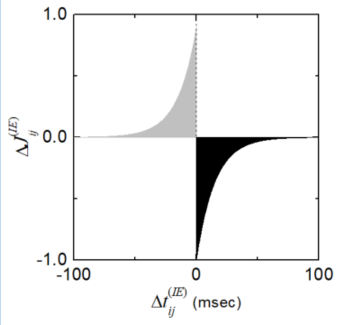



**- Anti-Hebbian E to I eSTDP**

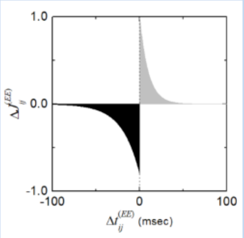
$$\Delta J_{ij}^{(IE)} = \begin{cases} -A_+ \exp(-\Delta t_{ij}^{(IE)} / \tau_+) & \text{for } \Delta t_{ij}^{(IE)} > 0 \\ A_- \exp(\Delta t_{ij}^{(IE)} / \tau_-) & \text{for } \Delta t_{ij}^{(IE)} < 0 \end{cases}$$

$\Delta t_{ij}^{(IE)} > 0 \rightarrow$ eLTD, $\Delta t_{ij}^{(IE)} < 0 \rightarrow$ eLTP

$A_+ = 1.0$
$A_- = 0.9$
$\tau_+ = 15.0$
$\tau_- = 15.0$
$\delta = 0.05$



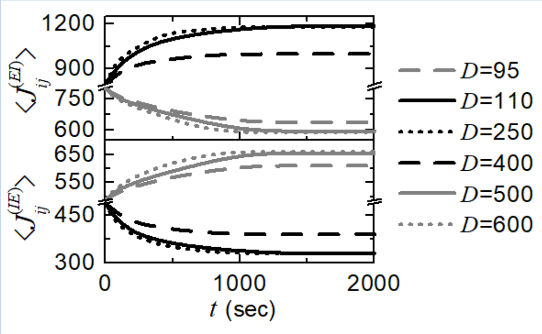

(cf. E to E eSTDP: Hebbian STDP)



# Long-term Potentiation (LTP) and Depression (LTD)

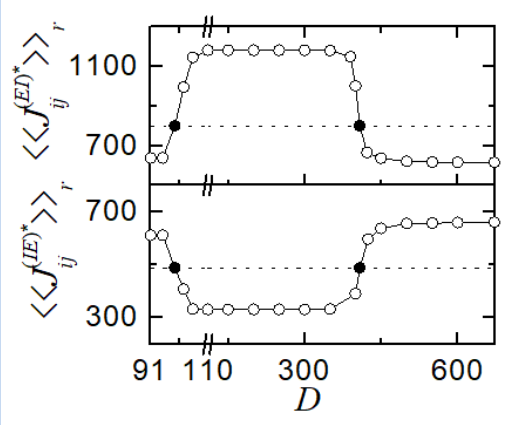
- **Fast Sparsely Synchronization in the Absence of STDP**
  Occurrence of Fast Sparsely Synchronization in the range of $D_1^* (\simeq 91) < D < D_2^* (\simeq 537)$

- **Time-Evolution of Population-Averaged Synaptic Strength $<J_{ij}^{(EI)}>$ & $<J_{ij}^{(IE)}>$**
  $D = 110, 250, 400$ (intermediate $D$): Monotonic increase (decrease) in $<J_{ij}^{(EI)}>$ ($<J_{ij}^{(IE)}>$) above $J_0^{(EI)}$ (below $J_0^{(IE)}$) and saturated to limit value → iLTP (eLTD)
  $D = 95, 500, 600$: (small & large $D$) Monotonic decrease (increase) in $<J_{ij}^{(EI)}>$ ($<J_{ij}^{(IE)}>$) below $J_0^{(EI)}$ (above $J_0^{(IE)}$) and saturated to limit value → iLTD (eLTP)




- **Population-Averaged Saturated Limit Values of Synaptic Strengths $<<J_{ij}^{(IE)*}>>_r$ & $<<J_{ij}^{(IE)*}>>_r$**

Occurrence of iLTP & eLTD in an intermediate region
  $[\widetilde{D}_l(\simeq 99) < D < \widetilde{D}_h(\simeq 408)]$:
  $<<J_{ij}^{(IE)*}>>_r$: Increase & $<<J_{ij}^{(IE)*}>>_r$: Decrease
Otherwise, occurrence of iLTD & eLTP in the regions of small & large $D$ :
  $<<J_{ij}^{(IE)*}>>_r$: Decrease & $<<J_{ij}^{(IE)*}>>_r$: Increase
$<<J_{ij}^{(IE)*}>>_r$: Bell-shaped graph. $<<J_{ij}^{(IE)*}>>_r$: Well-shaped graph.



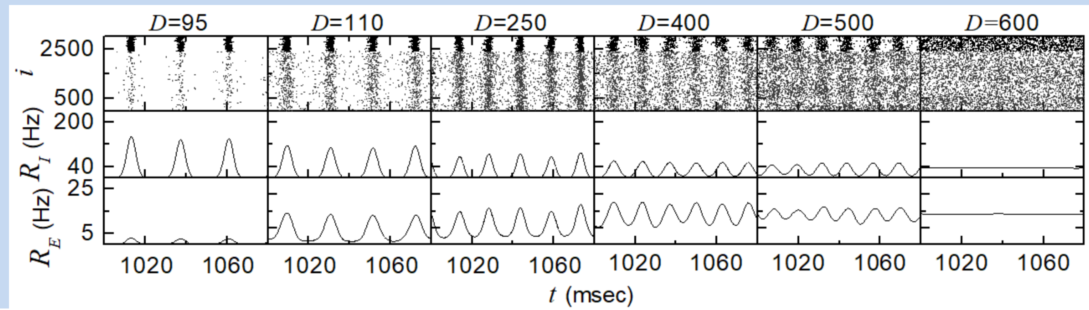
# Effect of the Interpopulation STDPs on the Fast Sparsely Synchronization

- **Raster Plots of Spikes and Instantaneous Population Spike Rates $R_X$ ($X = E$ or $I$)**

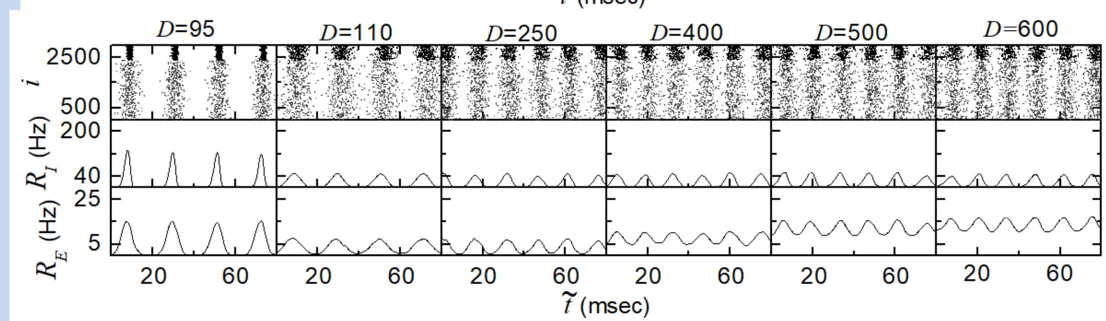
  Fast Sparsely Synchronization → Successive appearance of sparse spiking stripes in the raster plot of spikes & oscillating instantaneous population spike rate $R_X(t)$

Black: I-population, Gray: E-population

Absence of STDP




Presence of interpopulation STDPs




$D = 110, 250, 400$ (intermediate $D$)

Decrease in degree of Fast Sparsely Synchronization (Decrease in amplitudes of $R_X$)

Due to increased I to E synaptic inhibition (iLTP) and decreased E to I synaptic excitation (eLTD)

$D = 95, 500, 600$: (small & large $D$)

Increase in degree of Fast Sparsely Synchronization (Increase in amplitudes of $R_X$)

Due to decreased I to E synaptic inhibition (iLTD) and increased E to I synaptic excitation (eLTP)

# Equalization Effect in Interpopulation Synaptic Plasticity

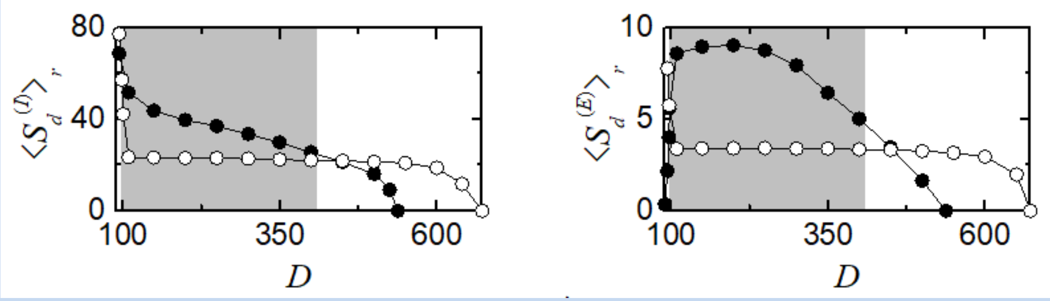
- **Characterization of Population Behaviors for Fast Sparsely Synchronization**

  Synchronization degree $S_d^{(X)}$: Time-averaged amplitude of $R_X(t)$

  Intermediate $D$ region (iLTP & eLTD: Gray region): Decrease in $S_d^{(X)}$

  Large & Small $D$ regions (iLTD & eLTP): Increase in $S_d^{(X)}$

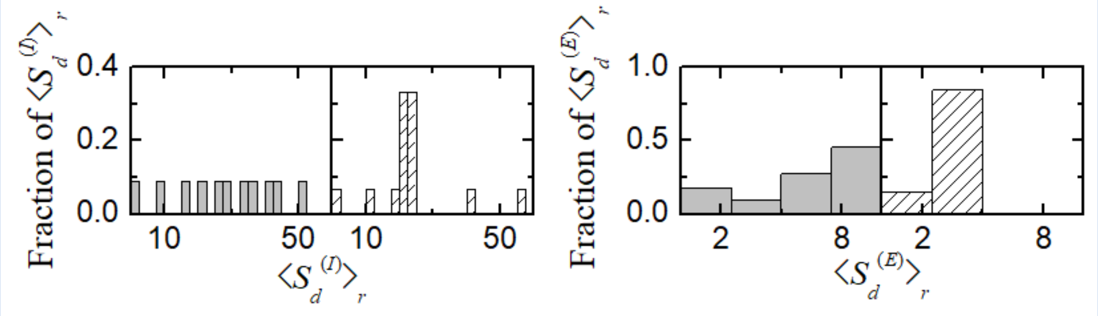
  $S_d^{(X)}$: Flat in a wide region of intermediate and large $D$ → Equalization effect




Open circles: Interpopulation STDPs
Solid circles: Absence of STDP

- **Equalization Effect in $S_d^{(X)}$**

  Equalization effect in $S_d^{(X)}$ with much smaller standard deviation

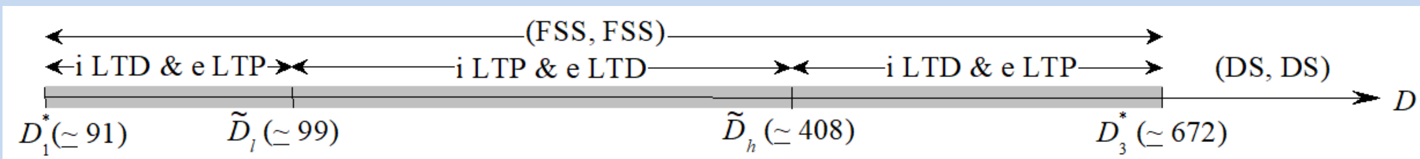



Left: Absence of STDP
Right: Interpopulation STDPs

# Summary

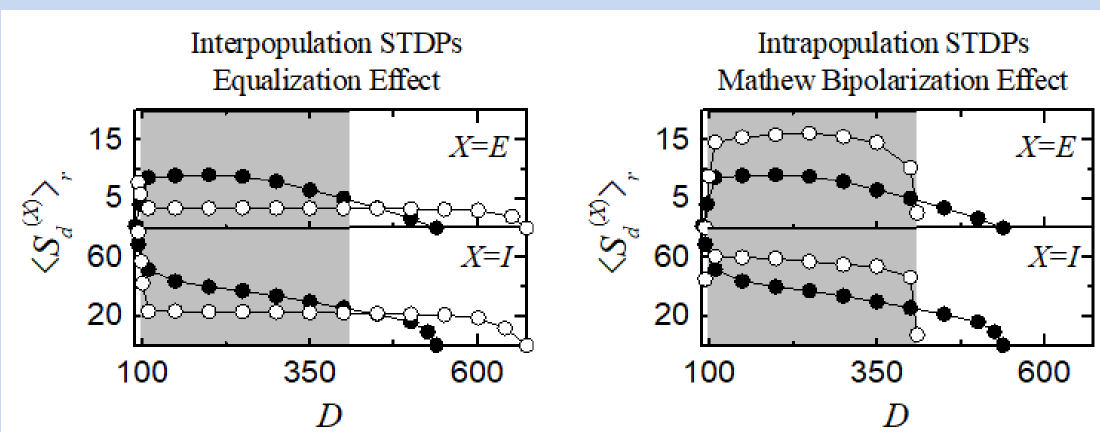
- **Fast Sparsely Synchronization in the Absence of STDP**
  - Fast Sparsely Synchronization (related to diverse cognitive functions) occurs in the clustered small-world networks with two inhibitory and excitatory populations.

- **Effect of Interpopulation (I to E & E to I) STDPs on the Fast Sparsely Synchronization**




  - Degree of good synchronization gets decreased, while degree of bad synchronization becomes increased.
  - Degree of fast sparsely synchronization becomes nearly the same in a wide range of noise intensity.

  → **Occurrence of Equalization Effect (also, occurrence of dumbing-down effect)**




cf. Matthew bipolarization effect in Intrapopulation (I to I & E to E) synaptic plasticity: Good (bad) synchronization becomes better (worse).


[1] S.-Y. Kim & W. Lim, Neural Netw. 106, 50 (2018).
[2] S.-Y. Kim & W. Lim, Neural Netw. 97, 92 (2018).

# Equalization Effect in Interpopulation Synaptic Plasticity

## • Characterization of Population Behaviors for FSS

FSS → Successive appearance of sparse spiking stripes in the raster plot of spikes

Average occupation degree $<O_i^{(X)}>$: Density of spikes in the spiking stripes

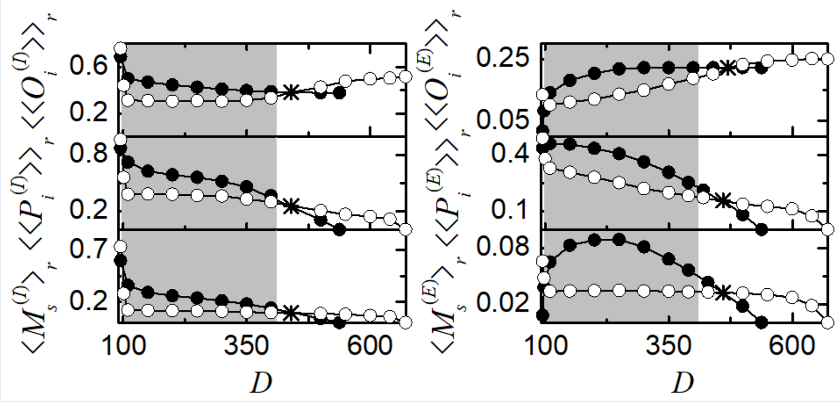
Average pacing degree $<P_i^{(X)}>$: Degree of phase coherence between spikes

Spiking measure $M_s^{(X)}$: Product of $<O_i^{(X)}>$ & $<P_i^{(X)}>$

Intermediate $D$ region (iLTP & eLTD: Gray region):
Decrease in $<O_i^{(X)}>$, $<P_i^{(X)}>$, & $M_s^{(X)}$

Large & Small $D$ regions (iLTD & eLTP):
Increase in $<O_i^{(X)}>$, $<P_i^{(X)}>$, & $M_s^{(X)}$

Open circles: Interpopulation STDPs
Solid circles: Absence of STDP

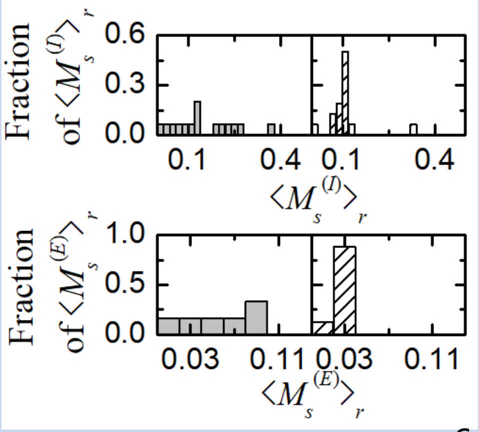



$<O_i^{(X)}>$: Relatively fast-increasing function
→ Non-equalization effect with larger standard deviation

$<P_i^{(X)}>$: Slowly-decreasing function → Weak equalization effect with smaller standard deviation

→ $M_s^{(X)}$: Flat in a wide region of intermediate and large $D$ (strong equalization effect)

Left: Absence of STDP
Right: Interpopulation STDPs




## • Strong Equalization Effect in $M_s^{(X)}$

Cooperative interplay between the weak equalization effect in decreasing $<P_i^{(X)}>$ and the non-equalization effect in increasing $<O_i^{(X)}>$

→ Strong equalization effect in $M_s^{(X)}$ with much smaller standard deviation