Modular and Global Sparse Synchronization in Clustered Small-World Networks of Inhibitory Fast Spiking Izhikevich Interneurons

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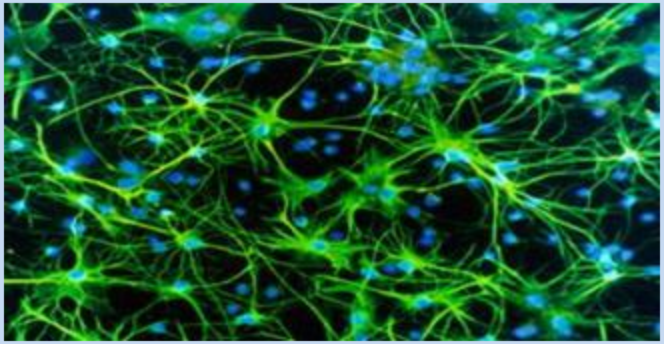

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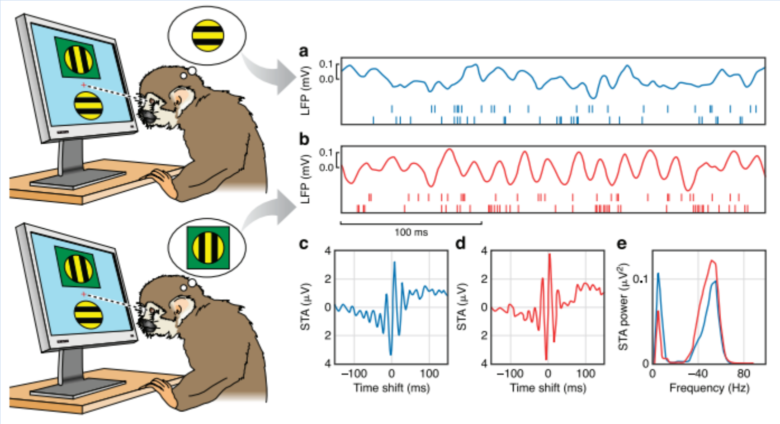
http://www.cnl.re.kr/

Behaving Brain Rhythms via Sparse Synchronization

- **Fast Sparsely Synchronized Brain Rhythms**
  - Population Level: Fast Oscillations
    e.g., gamma rhythm (30~100Hz) and sharp-wave ripple (100~200Hz)
  - Cellular Level: Stochastic and Intermittent Discharges
  - Associated with Diverse Cognitive Functions
    e.g., sensory perception, feature integration, selective attention, and memory formation and consolidation

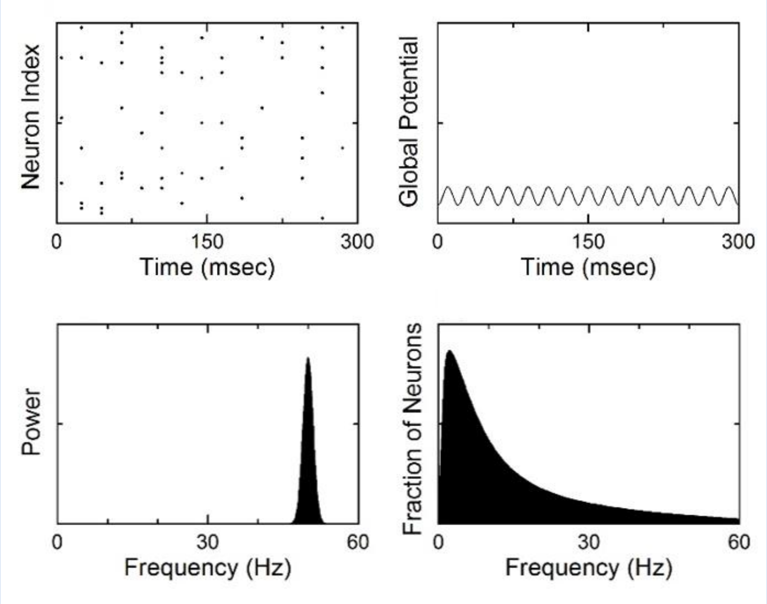


Gamma rhythm in visual cortex of behaving monkey

- **Sparsely Synchronized Brain Rhythms**

Individual Neurons: Intermittent and Stochastic Firings like Geiger Counters
Small-Amplitude Fast Population Rhythm via Sparse Synchronization of Individual Complex Firings

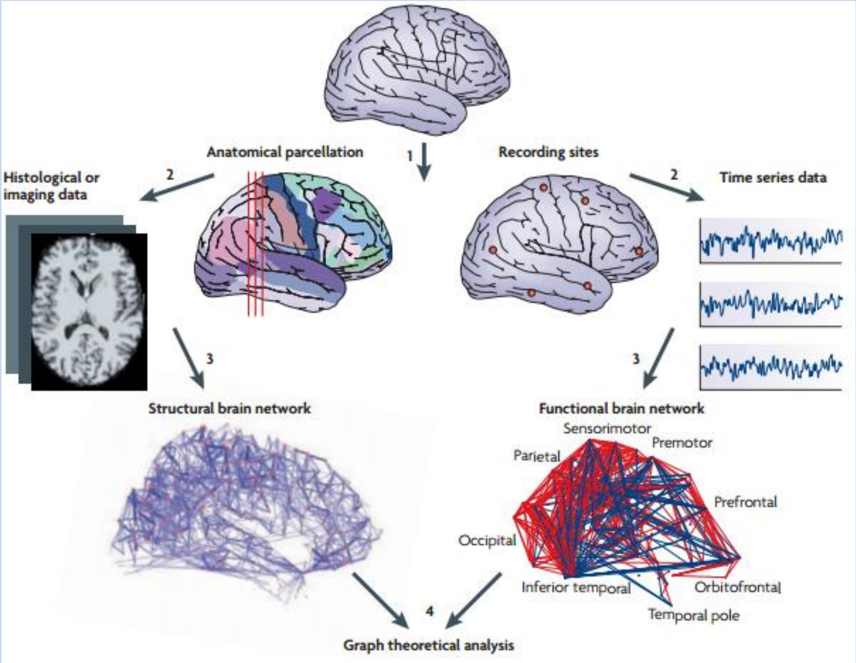
Coupled oscillators model: Inappropriate for investigation of the sparsely synchronized rhythms
→ **Coupled Subthreshold and/or Suprathreshold Neurons in the Presence of Strong Noise**
They exhibit noise-induced complex firing patterns



Complex Topology and Modular Structure of Real Brain

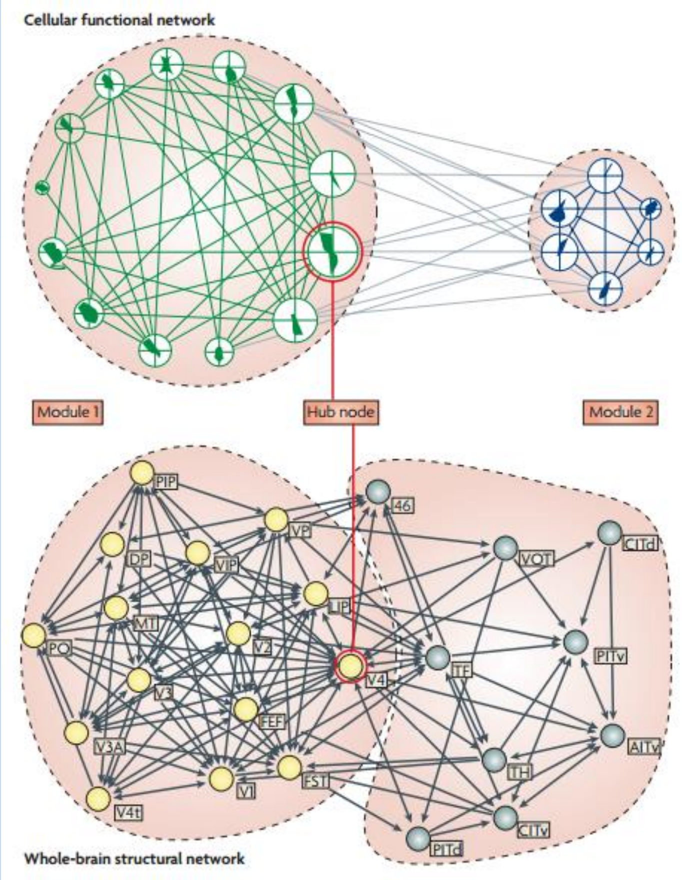
- **Complex Brain Network**

  Connection architecture of the real brain reveals complex topology such as small-worldness and scale-freeness which are neither regular nor random.



- **Modular Structure of Brain**

  The mammalian brain (e.g., cat and macaque) has been revealed to have a modular structure composed of sparsely linked clustered with spatial localization.



Brain Plasticity – Learning and Memory

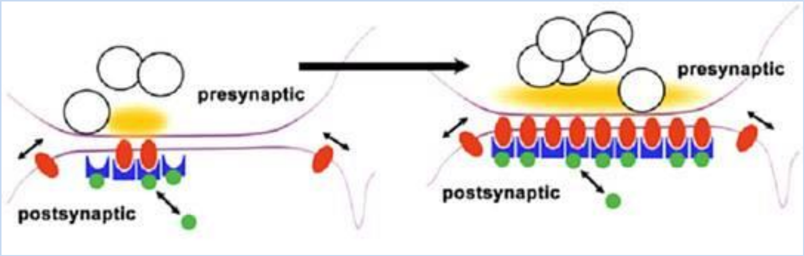
• Brain Plasticity

Brain plasticity refers to the brain's ability to change throughout life.

Brain plasticity occurs in the brain:
At the beginning of life, In case of brain injury, and
Through adulthood (whenever something new is learned and memorized)

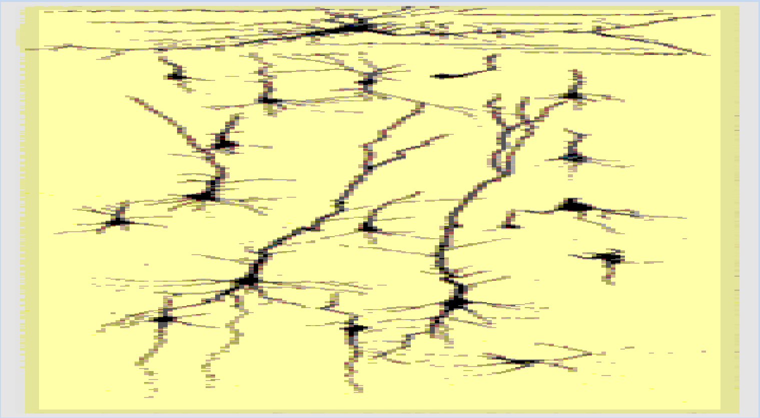
• Synaptic Plasticity

Synaptic plasticity is the ability of a synapse between neurons to change in strength over time.
→ **Change in the Strength of Synaptic Connections**



• Non-Synaptic Plasticity

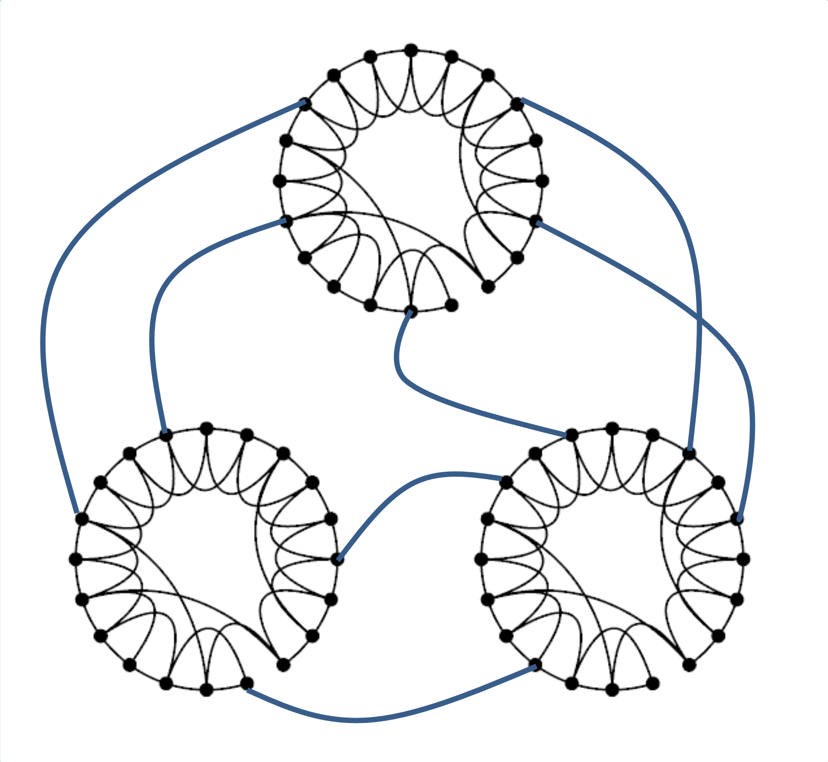
Non-synaptic plasticity involves modification of ion channel function in the axon, dendrites, and cell body.
→ **Change in the Synaptic Path Ways**



Sparse Synchronization in Clustered Small-World Networks

- **Clustered Small-World Network**

  Intra-modular connection: Small-World Network
  Inter-modular connection: Random



- **Interneuronal Network (I-I Loop)**

  Playing the role of the backbones of many brain rhythms by providing a synchronous oscillatory output to the principal cells

- **FS Izhikevich Interneuron**

  Izhikevich Interneuron Model: not only biologically plausible (Hodgkin-Huxley neuron-like), but also computationally efficient (IF neuron-like)

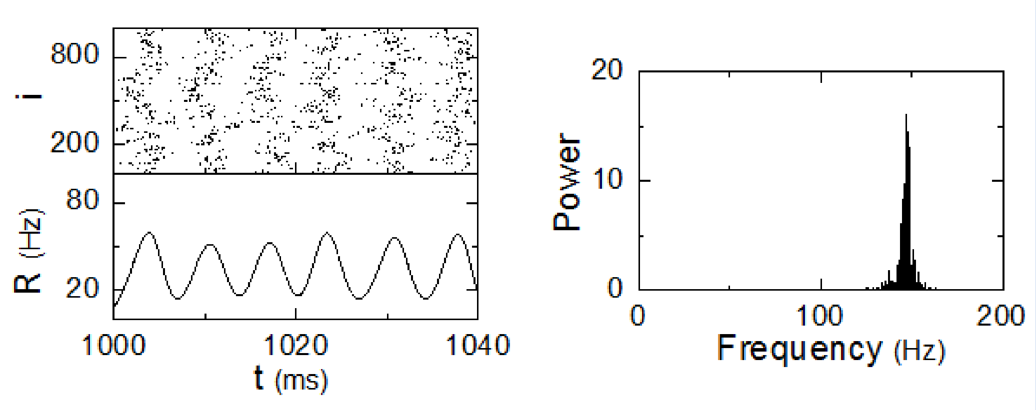
- **Effect of Inter-Modular Synaptic Connections**

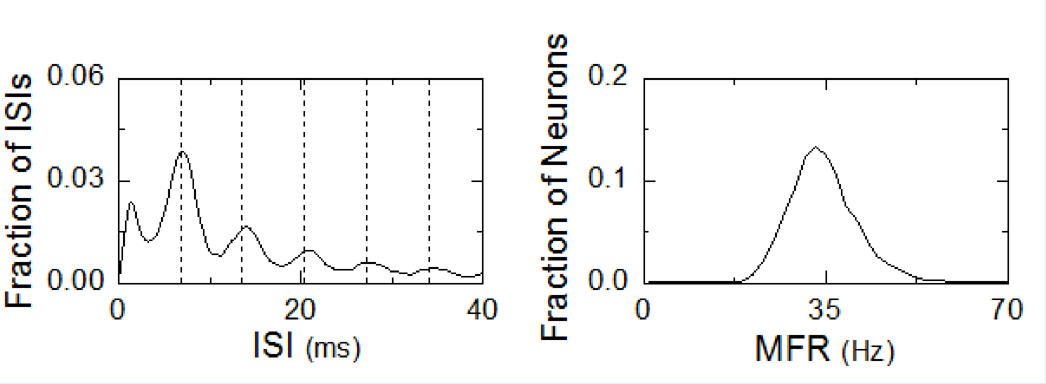
  Effect of Inter-Modular Synaptic Connection on **Sparsely-Synchronized Brain Rhythms** in Clustered Small-World Network
  ⇒ Implications for The Role of The Brain Plasticity

Fast Sparsely Synchronized Rhythms in Small-World Network

• Intra-Modular Dynamics

Fast Sparsely Synchronized State with the population frequency f_p=147 Hz and the individual neuron's mean firing rate f_i=33 Hz.





$$I_{DC} = 1500, M_{syn}^{(intra)} = 50, J_{intra} = 1400, D = 500, p_{rewiring} = 0.25, L = 10^3$$

Modular, Global Synchronization and Unsynchronization in Clustered Small-World Network

• Instantaneous Population Spike Rate

Instantaneous sub-population spike rates for the Ith sub-network:

$$R_S^{(I)}(t) \equiv \frac{1}{L}\sum_{i=1}^{L}\sum_{j=1}^{n_i^{(I)}} K_h(t - t_j^{(I,i)})$$

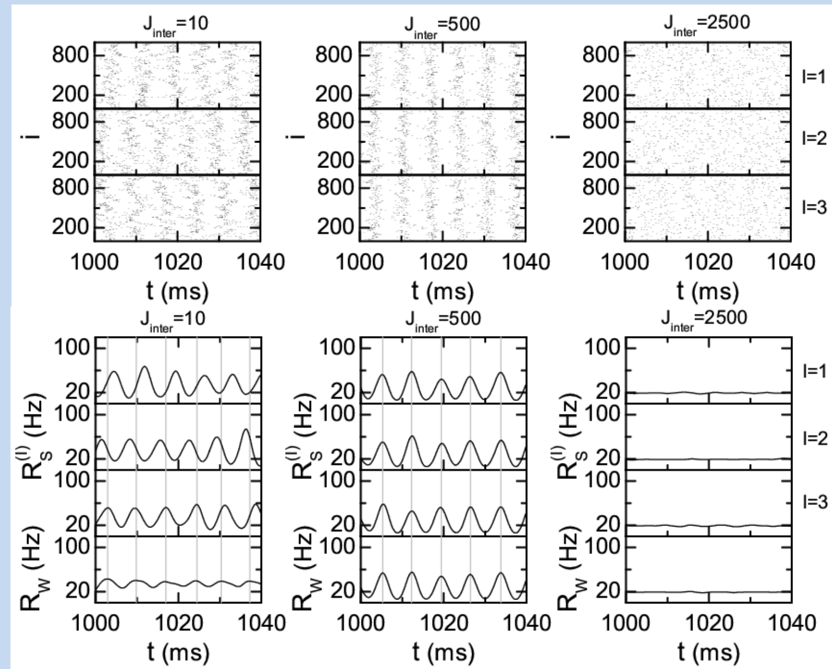
Gaussian kernel function of band width h:

$$K_h(t) = \frac{1}{\sqrt{2\pi}h} e^{-t^2/2h^2}, \qquad -\infty < t < \infty$$

Instantaneous whole-population spike rates for the whole network:

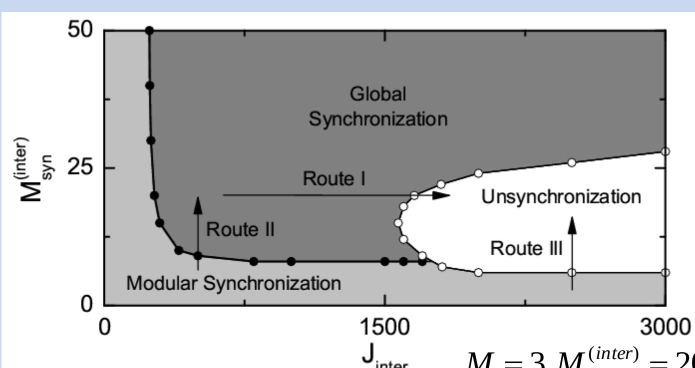
$$R_w(t) \equiv \frac{1}{M}\sum_{I=1}^{M} R_S^{(I)}(t) = \frac{1}{M \cdot L}\sum_{I=1}^{M}\sum_{i=1}^{L}\sum_{j=1}^{n_i^{(I)}} K_h(t - t_j^{(I,i)})$$

Modular Synchronization | Global Synchronization | Unsynchronization



$M = 3, M_{syn}^{(inter)} = 20$

• State Diagram in the $J_{inter} - M_{syn}^{(inter)}$ Plane



$M = 3, M_{syn}^{(inter)} = 20$

Modular Synchronization: Mismatching between modular synchronization of sub-networks.
Global Synchronization: Matching between modular synchronization of sub-networks
Unsynchronization

Synchronization-Unsynchronization Transition
Effect of Large Inter-modular Synaptic Strength

- **Realistic Thermodynamics Order Parameter**

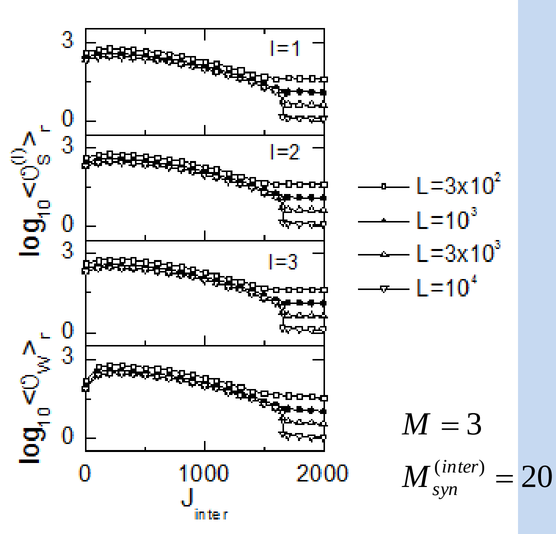
Sub-population order parameter for the *I*th sub-network:

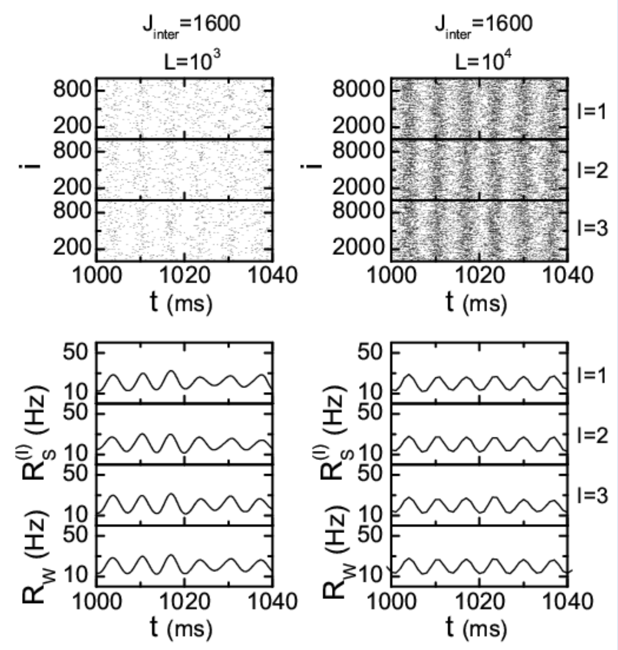
$$\mathcal{O}_S^{(I)} \equiv \overline{(R_S^{(I)}(t) - \overline{R_S^{(I)}(t)})^2}$$

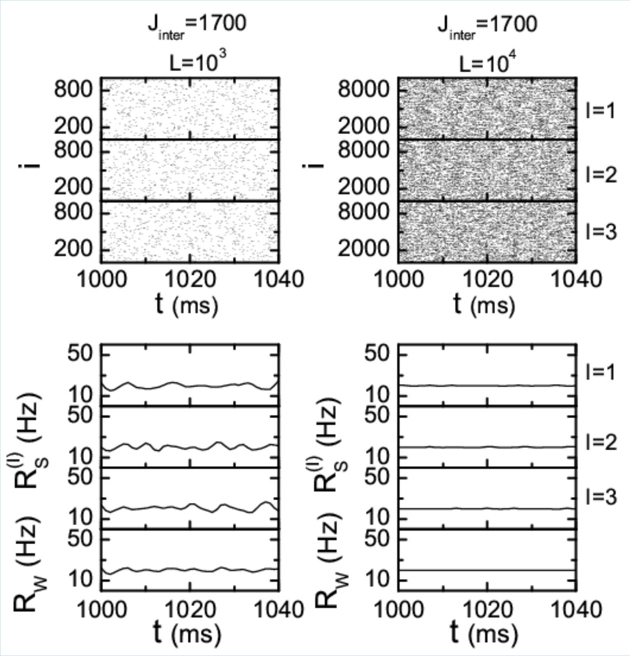
Whole-population order parameter for the whole network:

$$\mathcal{O}_W \equiv \overline{(V_W(t) - \overline{V_W(t)})^2}$$

For the synchronized state, $\mathcal{O}_S^{(I)}$ and $\mathcal{O}_W$ approach a non-zero limit value for N→∞.
For the unsynchronized state, $\mathcal{O}_S^{(I)}$ and $\mathcal{O}_W$ approach a zero limit value for N→∞.







Characterization of Synchronization and Unsynchronization Using the Spatial Cross-correlation

• Spatial Cross-Correlation

Instantaneous individual spike rate: $r_i^{(I)}(t) \equiv \sum_{j=1}^{n_i^{(I)}} K_h(t - t_j^{(I,i)})$

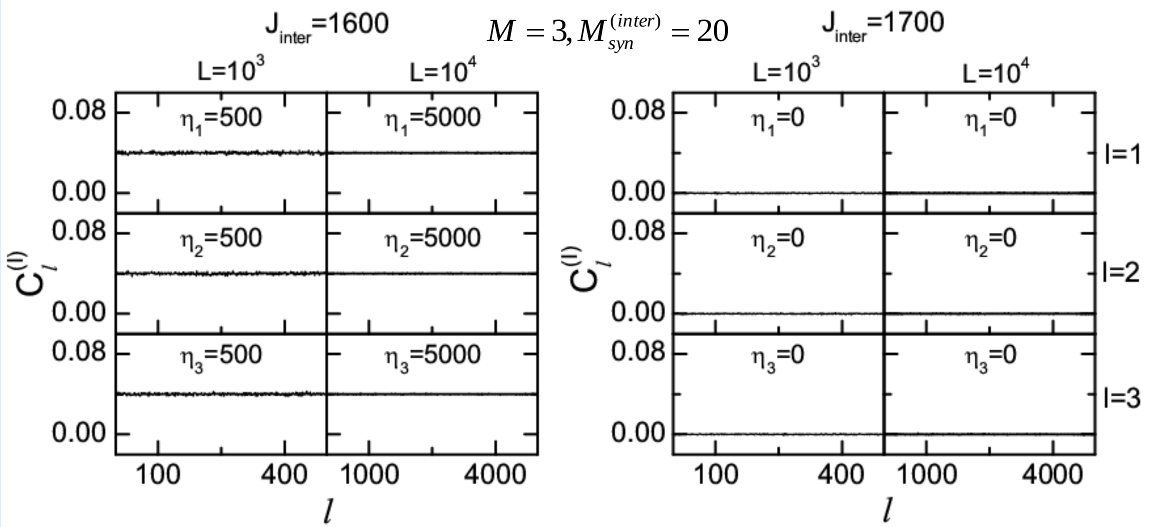
Normalized temporal cross-correlation function between in the instantaneous individual spike rate in the *I*th sub-network sub-population spike rates:

$$C_{i,j}^{(I)}(\tau) = \frac{\overline{\Delta r_i^{(I)}(t+\tau) \cdot \Delta r_j^{(I)}(t)}}{\sqrt{\overline{\Delta r_i^{(I)^2}(t)}}\sqrt{\overline{\Delta r_j^{(I)^2}(t)}}}$$

Spatial cross-correlation function: $C_l^{(I)} = \frac{1}{L}\sum_{i=1}^{L} C_{i,i+l}^{(I)}(0)$ for $L = 1, \ldots, L/2.$

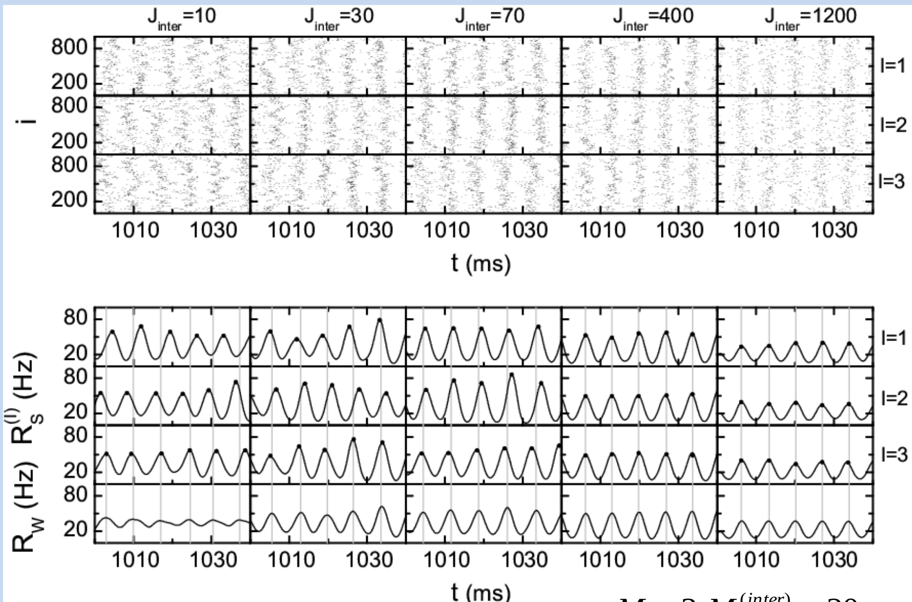
For synchronized state:
$C_l^{(I)}$: nearly non-zero constant for whole range of l.
Correlation length = L/2

For unsynchronized state:
$C_l^{(I)}$: nearly zero for whole range of l.
Correlation length = 0



Modular-Global Synchronization Transition
Effect of Small Inter-modular Synaptic Strength

• Modular and Global Synchronization

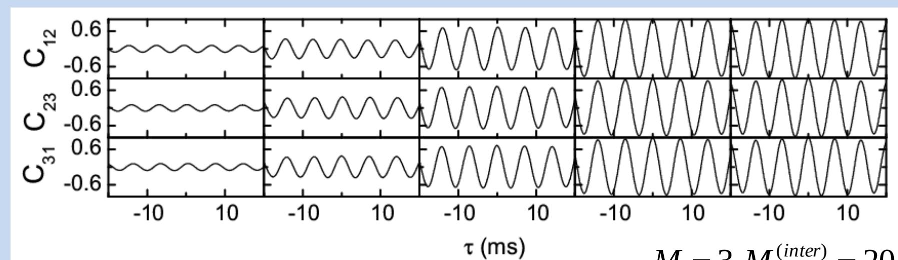


$M=3, M_{syn}^{(inter)}=20$

Normalized temporal cross-correlation function between the instantaneous sub-population spike rates:

$$C_{I,J}(\tau)=\frac{\overline{\Delta R_S^{(I)}(t+\tau)\cdot\Delta R_S^{(J)}(t)}}{\sqrt{\overline{\Delta R_S^{(I)2}(t)}}\sqrt{\overline{\Delta R_S^{(J)2}(t)}}}$$

$$\Delta R_S^{(I)}(t)=R_S^{(I)}(t)-\overline{R_S^{(I)}(t)}$$



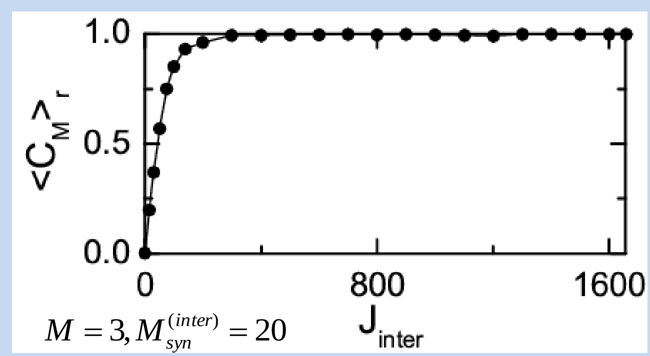
$M=3, M_{syn}^{(inter)}=20$

• Cross-Correlation Modularity Measure

$$C_M=\frac{2}{M(M-1)}\sum_{I=1}^{M}\sum_{J=I+1}^{M}C_{I,J}(0)$$

$J_{inter} < J_{inter}^{**}(\simeq 268)$: Modular Sync. : $0<\langle C_M\rangle_r<1$

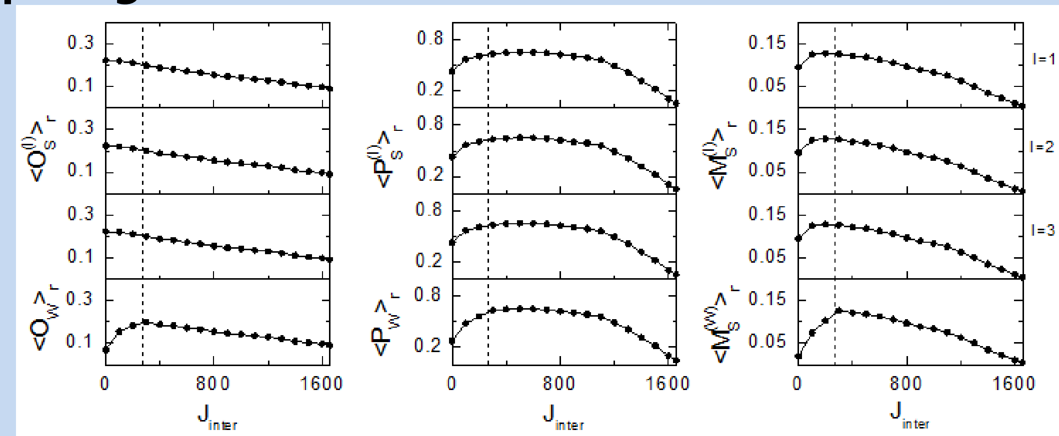
$J_{inter}^{**} < J_{inter} < J_{inter}^{*}$: Global Sync. : $\langle C_M\rangle_r \simeq 1$



Characterization of Degree of Synchronization

• Realistic Statistical-Mechanical Spiking Measure

$M = 3, M_{syn}^{(inter)} = 20$



Occupation degree: representing the density of stripe in the raster plot

Pacing degree: representing the smearing of stripe in the raster plot (average contribution of all microscopic spikes in the stripe)

With increasing J_{inter},

$\langle O_S^{(I)} \rangle_r$: decreases monotonically.

$\langle P_S^{(I)} \rangle_r$: exhibits the bell-shaped curve.

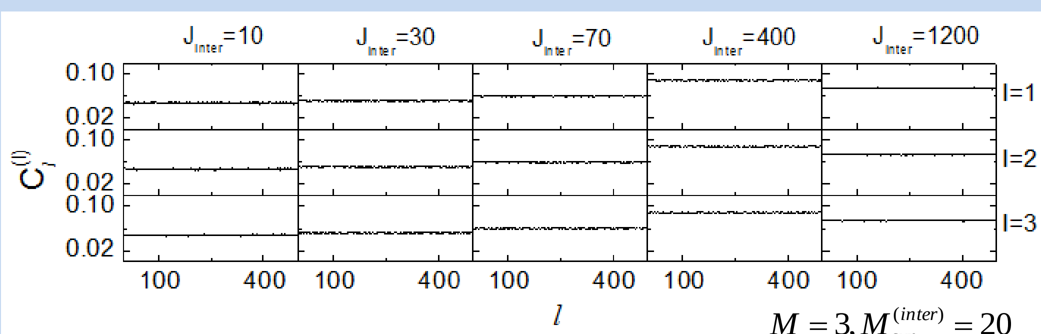
Modular Sync. : $\langle O_S^{(I)} \rangle_r < \langle O_W \rangle_r$, $\langle P_S^{(I)} \rangle_r < \langle P_W \rangle_r$, $\langle M_S^{(I)} \rangle_r < \langle M_S^{(W)} \rangle_r$

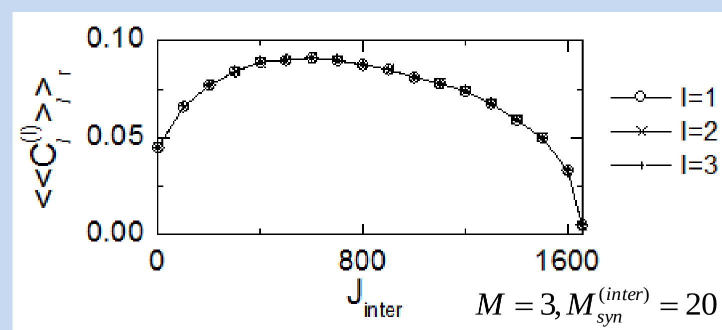
Global Sync. : $\langle O_S^{(I)} \rangle_r \simeq \langle O_W \rangle_r$, $\langle P_S^{(I)} \rangle_r \simeq \langle P_W \rangle_r$, $\langle M_S^{(I)} \rangle_r \simeq \langle M_S^{(W)} \rangle_r$

• Spatial Cross-Correlation Based Measure

Average value of spatial cross-correlation function: Same behavior with average pacing degree.

$\rightarrow$ Implication with the measure for the degree of synchronization





Summary

- **Investigation of The Effect of Inter-Connection on Emergence of Sparsely Synchronized Cortical Rhythms**

Occurrence of Modular Sparse Synchronization and Global Sparse Synchronization

Modular sparse synchronization: the population behavior reveals the clustering structure due to some mismatching between the intra-modular dynamics of the sub-networks

Global Sparse Synchronization: the population behavior is globally identical, independently of the cluster structure, because of the perfect matching between the intra-modular dynamics of sub-networks

Dual Roles of Inter-Modular Coupling Strength J_{inter} Depending on Its Strength:

For large J_{inter} $\rightarrow$ Destructive role to "spoil" the pacing between sparse spikings

For small J_{inter} $\rightarrow$ Constructive role to "favor" the pacing between spikings in each sub-network.

Role of Number of Inter-Modular Connection Probability:

Constructive role to "favor" global communication between sub-networks

Important implications for the role of the **Brain Plasticity** which refers to the brain's ability to change its structure and function by modifying the strength or efficacy of synaptic transmission.