Effect of Small-World Connectivity on Fast Sparsely Synchronized Cortical Rhythms

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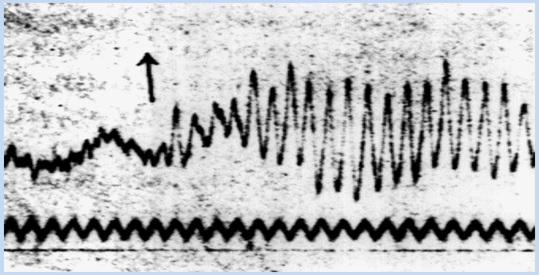
- **Slow Brain Rhythms for the Silent Brain**

Alpha Rhythm

[H. Berger, Arch. Psychiatr Nervenkr.87, 527 (1929)]

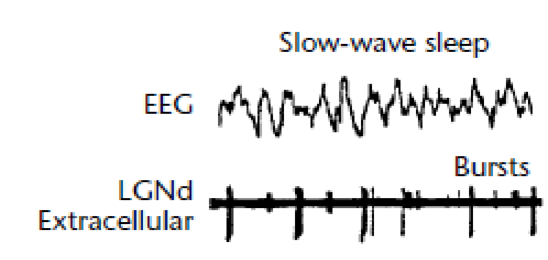
Slow brain rhythm (3~12Hz) with large amplitude during the contemplation with closing eyes



Sleep Spindle Rhythm

[M. Steriade, et. Al. J. Neurophysiol. 57, 260 (1987).]

Brain rhythm (7~14Hz) with large amplitude during deep sleep without dream

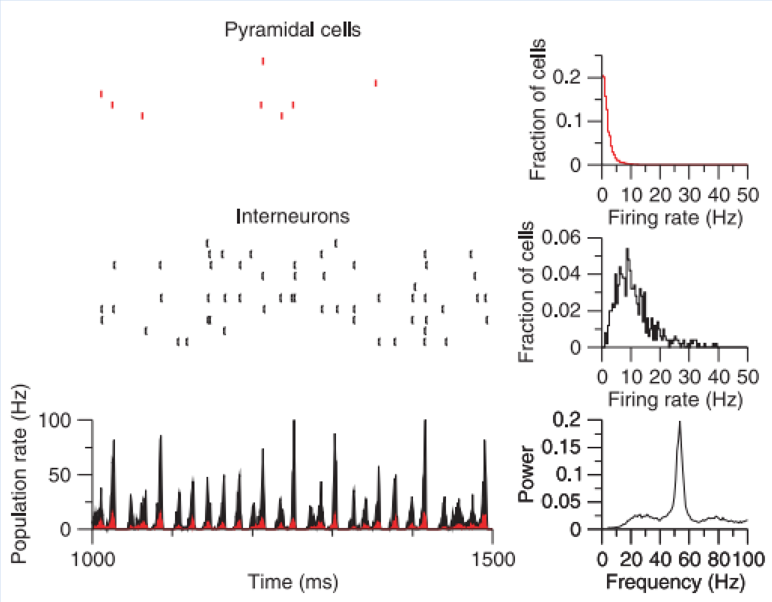


Fast Sparsely Synchronized Cortical Rhythms

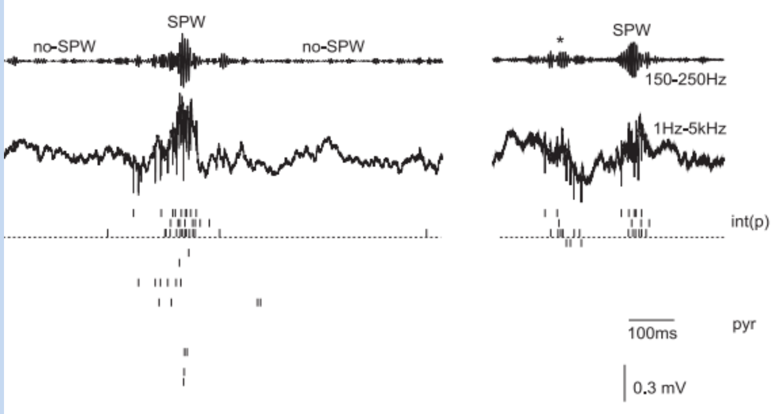
- **Gamma Rhythm (30-100 Hz) in the Awake Behaving States**

Fast Small-Amplitude Population Rhythm (55 Hz) with Stochastic and Intermittent Neural Discharges (Interneuron: 2 Hz & Pyramidal Neuron: 10 Hz)

Associated with Diverse Cognitive Functions (sensory perception, feature integration, selective attention, and memory formation)

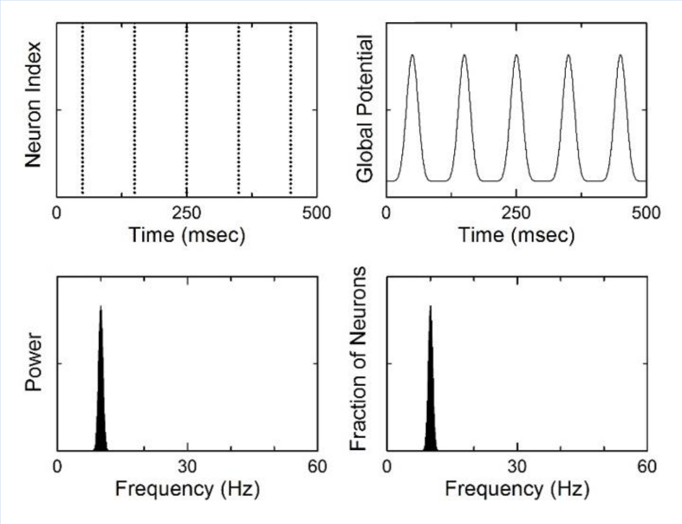


- **Sharp-Wave Ripples (100-200 Hz)**
  - Sharp-Wave Ripples in the Hippocampus
    Appearance during Slow-Wave Sleep
    (Associated with Memory Consolidation)
  - Sharp-Wave Ripples in the Cerebellum
    Millisecond Synchrony between Purkinje Cells
    → Fine Motor Coordination



Sparse Synchronization vs. Full Synchronization

- **Fully Synchronized Rhythms**

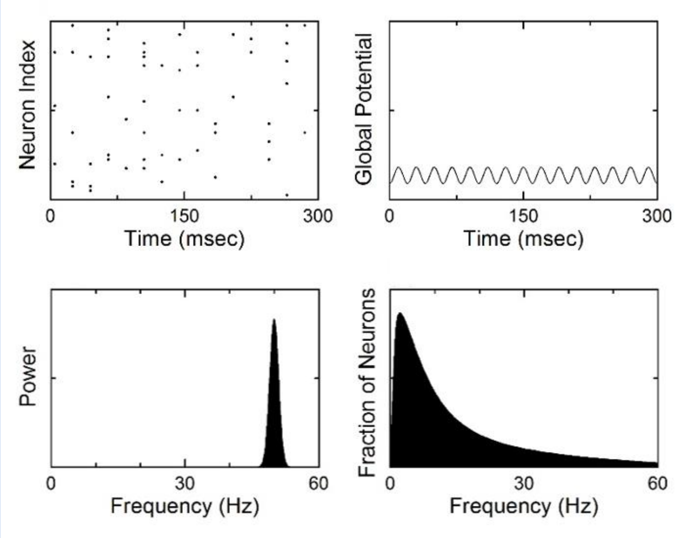


Individual Neurons: Regular Firings like Clocks

Large-Amplitude Population Rhythm via **Full Synchronization** of Individual Regular Firings

Investigation of This Huygens Mode of Full Synchronization Using the **Conventional Coupled (Clock-Like) Oscillator Model**

- **Sparsely Synchronized Rhythms**



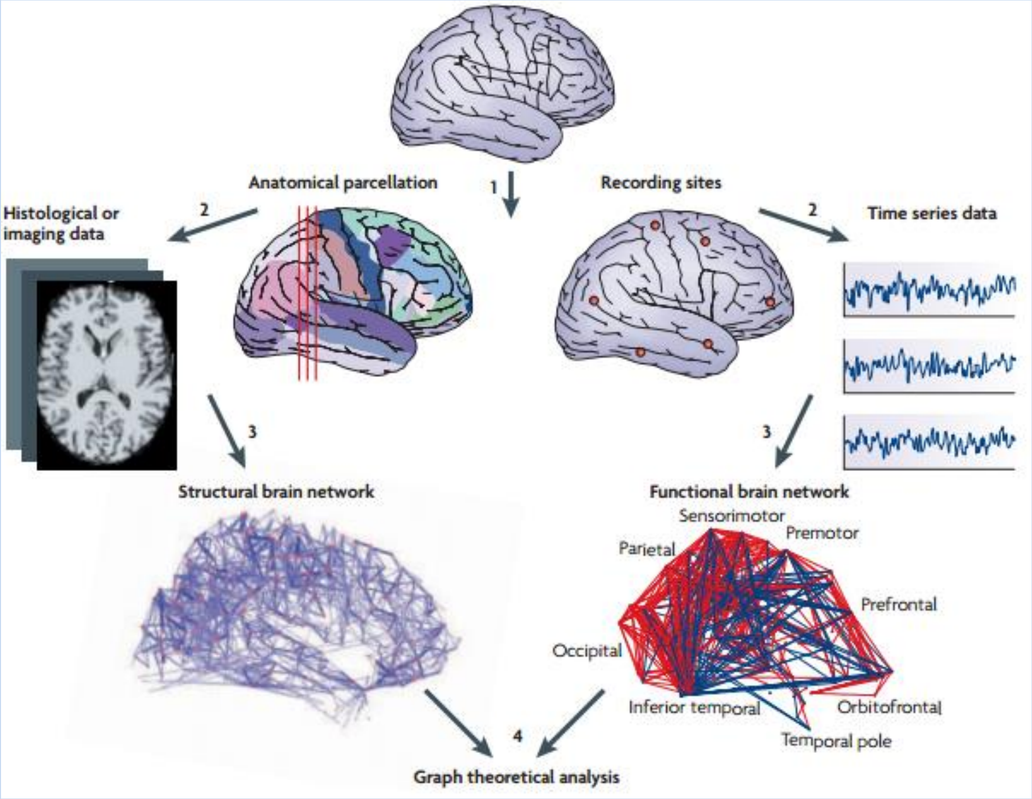
Individual Neurons: Intermittent and Stochastic Firings like Geiger Counters

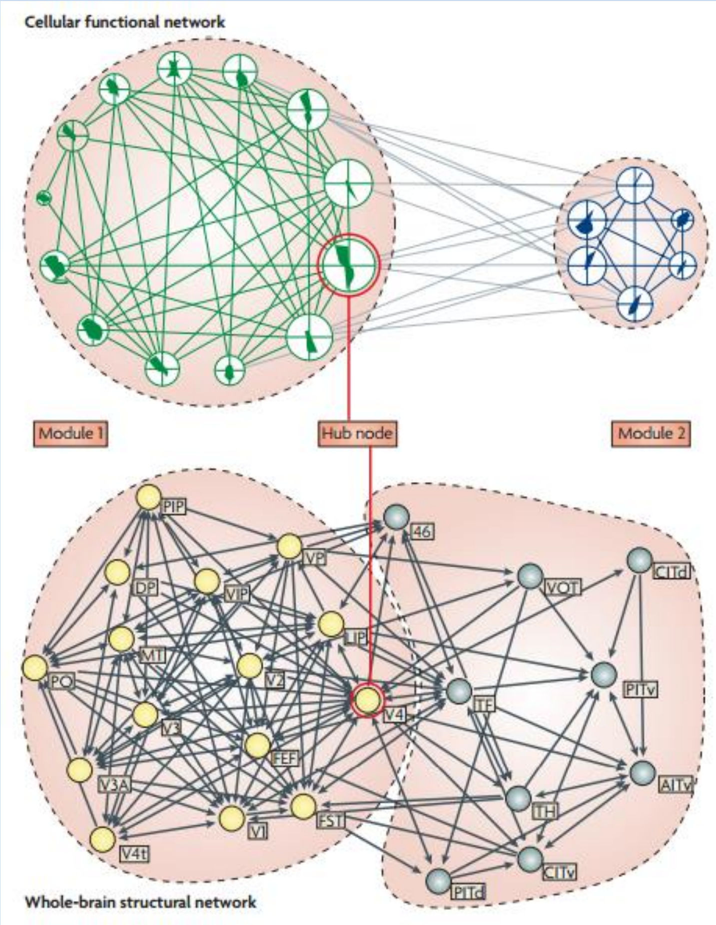
Small-Amplitude Fast Population Rhythm via **Sparse Synchronization** of Individual Complex Firings

Investigation of Sparse Synchronization in Networks of **Coupled (Geiger-Counter-Like) Neurons Exhibiting Complex Firing Patterns**

Complex Brain Network

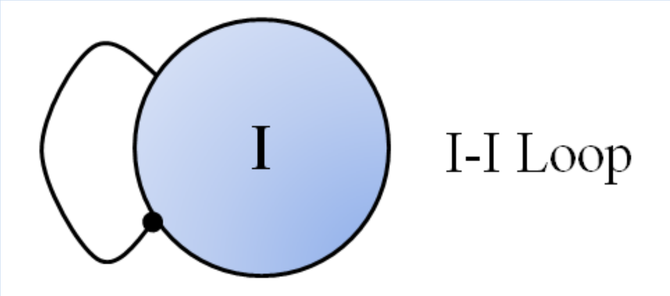
Network Topology: Complex (Neither Regular Nor Random)





Network of Inhibitory Fast-Spiking (FS) Izhikevich Interneurons

- **Interneuronal Network (I-I Loop)**



Playing the role of the backbones of many brain rhythms by providing a synchronous oscillatory output to the principal cells

- **FS Izhikevich Interneuron**

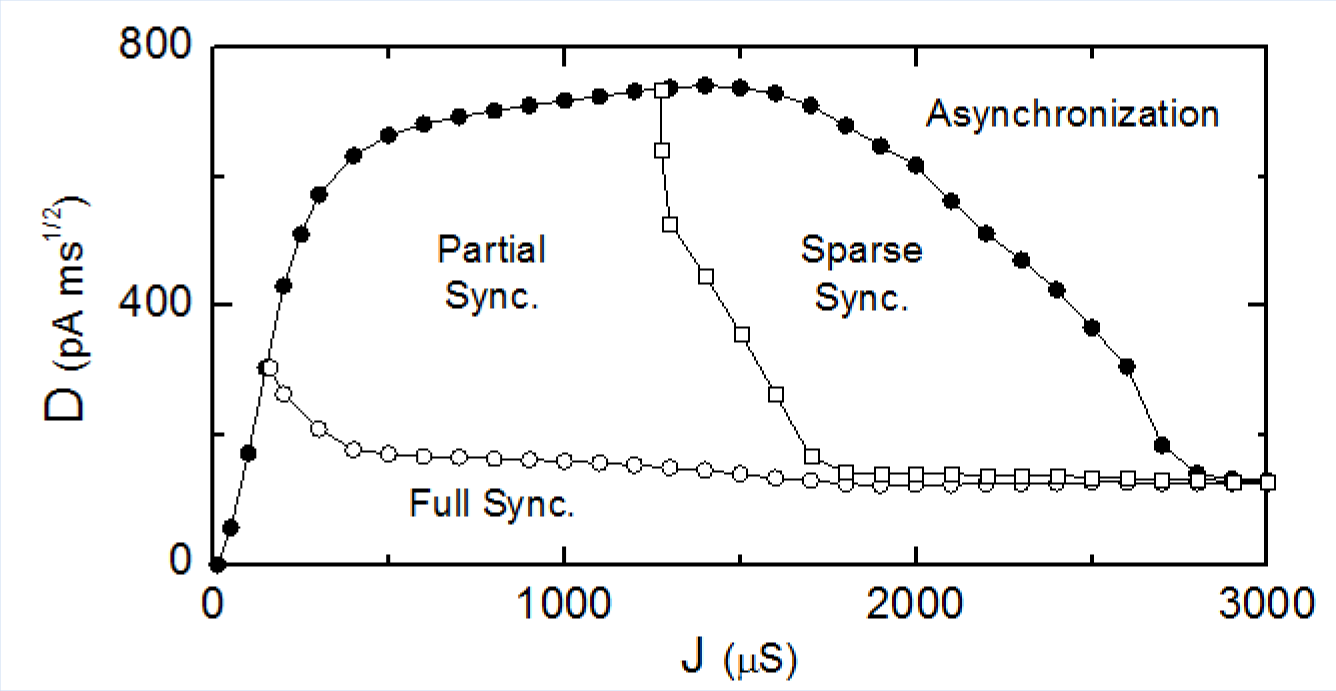
Izhikevich Interneuron Model: not only biologically plausible (Hodgkin-Huxley neuron-like),
but also computationally efficient (IF neuron-like)

Population Synchronization in the Random Network of FS Izhikevich Interneurons

- **Conventional Erdös-Renyi (ER) Random Graph**

  Complex Connectivity in the Neural Circuits: Modeled by Using The ER Random Graph for M_{syn}=50

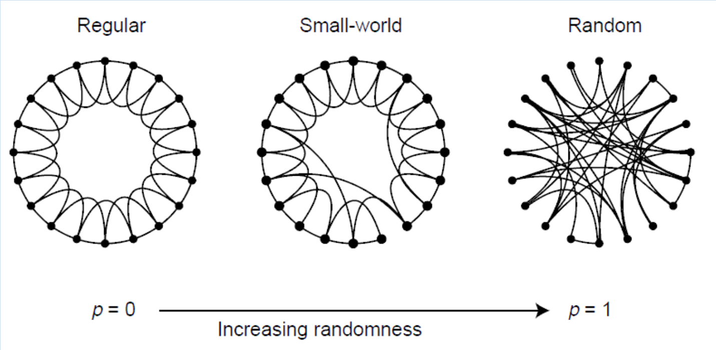
- **State Diagram in the J-D Plane for I_{DC}=1500**



Emergence of Sparsely Synchronized Rhythms in a Small World Network of FS Interneurons

Cortical Circuits: Neither Regular Nor Random

- **Watts-Strogatz Small World Network**



Interpolating between the Regular Lattice and the Random Graph via Rewiring

Start with directed regular ring lattice with N neurons where each neuron is coupled to its first k neighbors. Rewire each outward connection at random with probability p such that self-connections and duplicate connections are excluded.

- **Asynchrony-Synchrony Transition**

Investigation of Population Synchronization by Increasing the Rewiring Probability p for J=1400 & D=500

Thermodynamic Order Parameter:

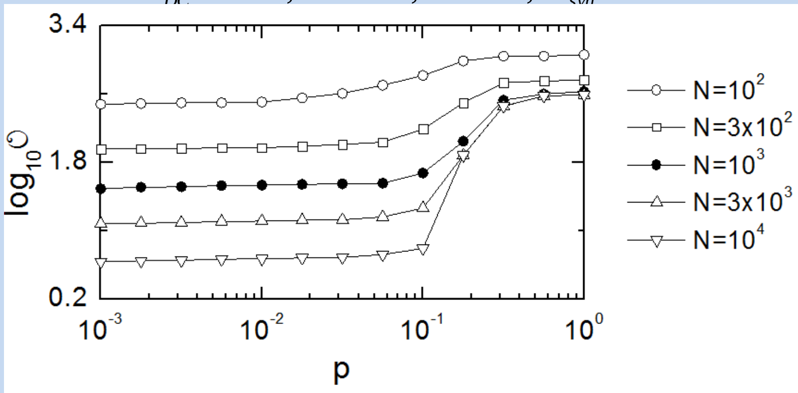
$$\mathcal{O} \equiv \overline{(\Delta V_G)^2} = \overline{(V_G(t) - \overline{V_G(t)})^2}$$

$$V_G(t) = \frac{1}{N}\sum_{i=1}^{N} v_i(t)$$

(Population-Averaged Membrane Potential)

Incoherent State: $N \to \infty$, then $\mathcal{O} \to 0$
Coherent State: $N \to \infty$, then $\mathcal{O} \to$ Non-zero value

$I_{DC} = 1500, J = 1400, D = 500, M_{syn} = 50$



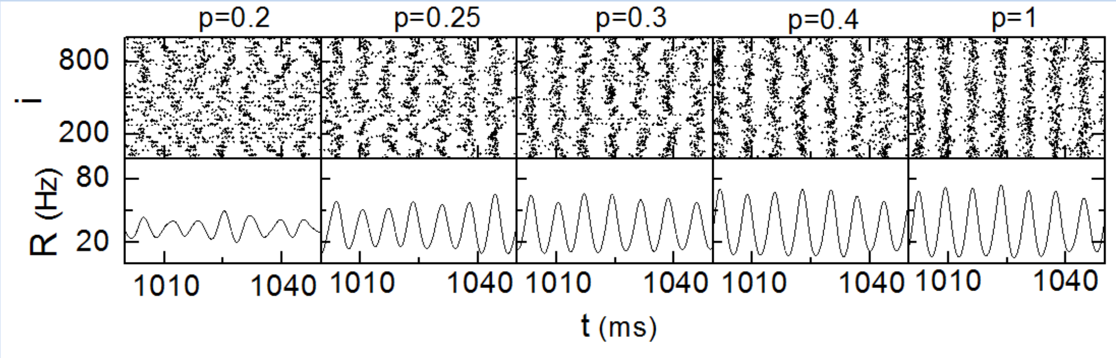
Occurrence of Population Synchronization for $p > p_{th}$ ($\simeq$ 0.12)

Characterization of Sparsely Synchronized States

- **Raster Plot and Global Potential**

  With increasing p, the zigzagness degree in the raster plot becomes reduced.
  $p > p_{max}$ (~0.4): Raster plot composed of stripes without zigzag and nearly same pacing degree. Amplitude of V_G increases up to p_{max}, and saturated.
  Appearance of Ultrafast Rhythm with f_p = 147 Hz

$I_{DC} = 1500, J = 1400, D = 500, N = 10^3, M_{syn} = 50$

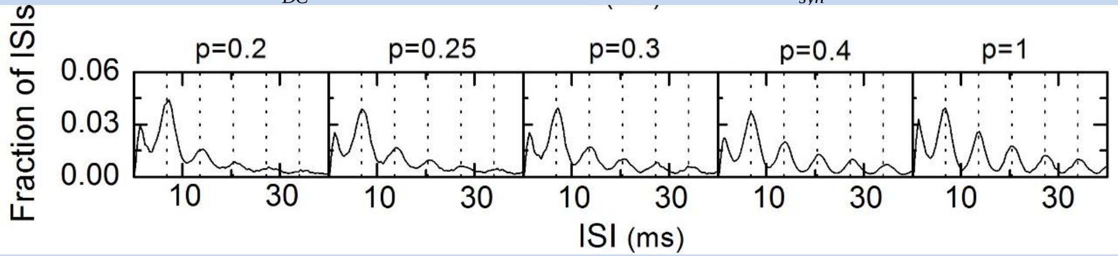


- **Interspike Interval Histograms**

  Multiple peaks at multiples of the period of the global potential

  Stochastic Phase Locking Leading to Stochastic Spike Skipping

$I_{DC} = 1500, J = 1400, D = 500, N = 10^3, M_{syn} = 50$



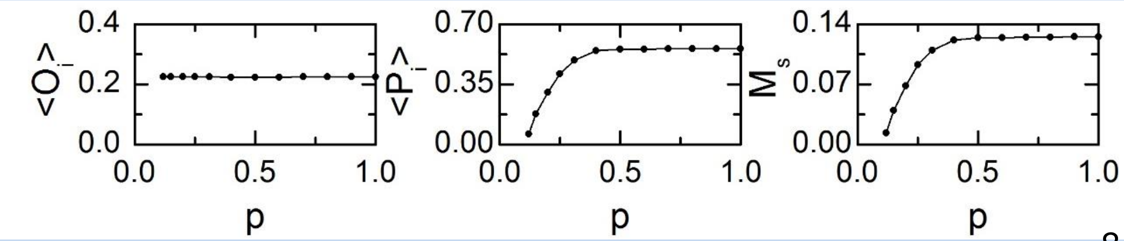
- **Statistical-Mechanical Spiking Measure**

  Taking into Consideration the Occupation (O_i) and the Pacing Degrees (P_i) of Spikes in the Stripes of the Raster Plot

  $\rightarrow M_i = O_i \times P_i$

  $\rightarrow M_s = \frac{1}{N_s} \sum_{i=1}^{N_s} M_i, \quad N_s$: No. of stripes

$I_{DC} = 1500, J = 1400, D = 500, N = 10^3, M_{syn} = 50$



Investigation of Population Synchronization in Terms of Spatial Correlation

- **Spatial Cross-Correlation**

Instantaneous individual spike rate

$$r_i(t) = \sum_{s=1}^{n_i} K_h(t - t_s^{(i)})$$

Cross-correlation function between r_i and r_j

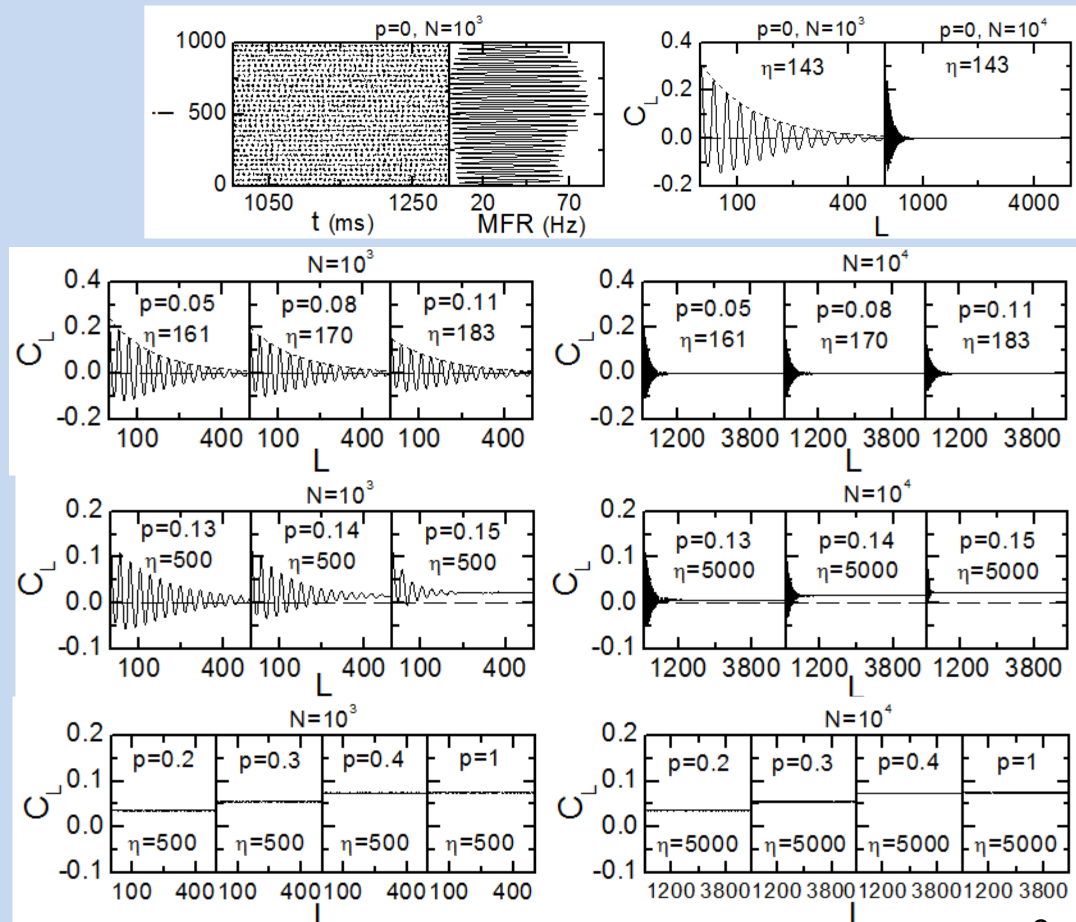
$$C_{i,j}(\tau) = \frac{\overline{\Delta r_i(t+\tau)\Delta r_j(t)}}{\sqrt{\overline{\Delta r_i^2(t)}}\sqrt{\overline{\Delta r_j^2(t)}}}$$

Spatial cross-correlation: $C_L = \frac{1}{N}\sum_{i=1}^{N} C_{i,i+L}(0)$

Horizontal stripe in the raster plot for p=0
→ Regular oscillation of MFR
→ Damped oscillation in C_L

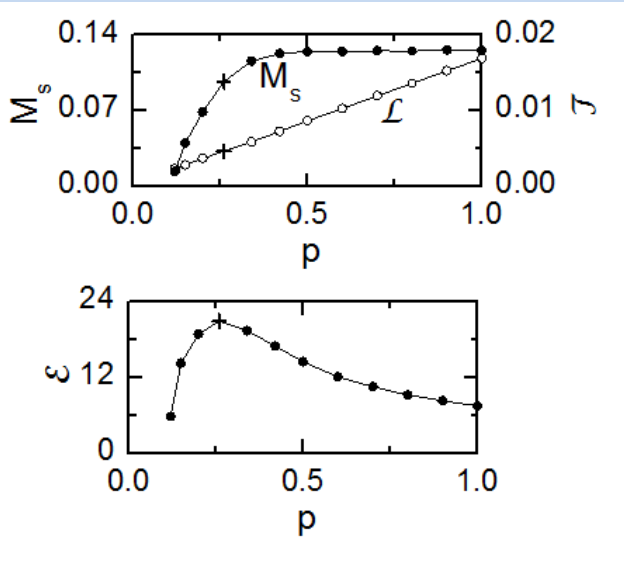
For the unsynchronization,
C_L makes an oscillator decay to zero.
As N increased, the normalized correlation length $\tilde{\eta}(=\eta/N)$ tends to zero.
→ No Global Synchronization Occurs.

For the synchronization,
C_L becomes non-zero constant.
Correlation length becomes N/2 covering the whole system.
→ Whole system is composed of just one single synchronized block.
→ Global synchronization occurs.



Economic Small-World Network

$I_{DC} = 1500, J = 1400, D = 500, N = 10^3, M_{syn} = 50$



- **Synchrony Degree M_s and Wiring Length λ**

  With increasing p, synchrony degree M_s is increased until $p=p_{max}$ because global efficiency of information transfer becomes better.

  Wiring length increases linearly with respect to p.
  → With increasing p, the wiring cost becomes expensive.

- **Dynamical Efficiency Factor**

  Tradeoff between Synchrony and Wiring Economy

$$\eta(p) = \frac{\text{Synchrony Degree}}{\text{Normalized Wiring Length}}$$

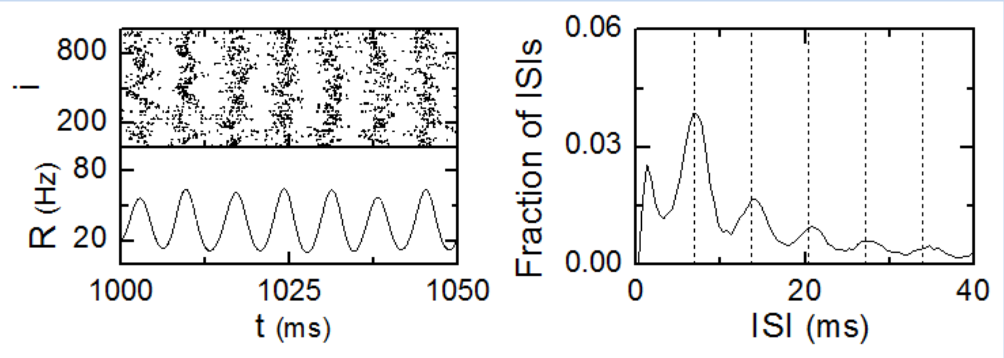
- **Optimal Sparsely-Synchronized Rhythm for $p=p^*_{DE}$**

  Optimal Ultrafast Rhythm Emerges at A Minimal Wiring Cost in An Economic Small-World Network for $p=p^*_{DE}$ ($\simeq$0.31).

$I_{DC} = 1500, J = 1400, D = 500, p = 0.31, N = 10^3, M_{syn} = 50$

  Optimal Sparsely-Synchronized Ultrafast Rhythm for $p=p^*_{DE}$ ($\simeq$ 0.31)

  Raster plot with a zigzag pattern due to local clustering of spikes
  Regular oscillating global potential



Summary

- **Emergence of Fast Sparsely Synchronized Rhythm in A Small-World Network of Inhibitory Izhikevich FS Interneurons**

Regular Lattice of Izhikevich FS Interneurons (p=0)
→ Unsynchronized Population State

Occurrence of Ultrafast Sparsely Synchronized Rhythm as the Rewiring Probability Passes a Threshold p_{th} (~0.12):
→ Population Rhythm ~ 147 Hz (Small-Amplitude Ultrafast Sinusoidal Oscillation)

Intermittent and Irregular Discharge of Individual Interneurons at 33 Hz
(Geiger-Counter-Like Firings)

Emergence of Optimal Ultrafast Sparsely-Synchronized Rhythm at A Minimal Wiring Cost in An Economic Small-World Network for $p=p^*_{DE}$ (~0.31)

$$I_{DC} = 1500, J = 1400, D = 500, p = 0.31, N = 10^3, M_{syn} = 50$$