Noise-Induced Burst and Spike Synchronizations in An Inhibitory Small-World Network of Subthreshold Bursting Neurons

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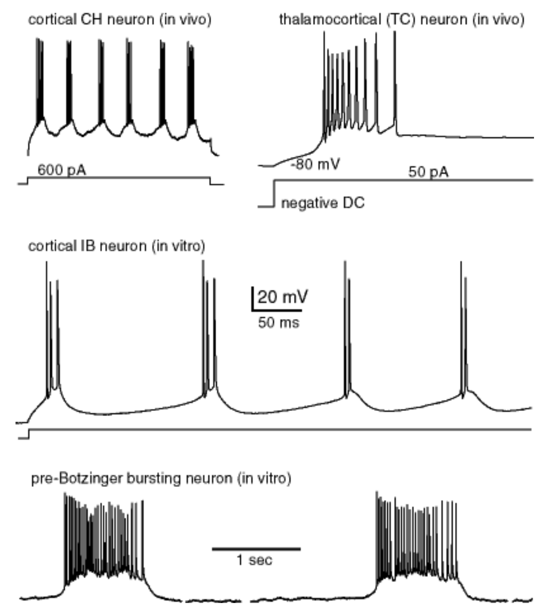
• Burstings with the Slow and Fast Time Scales

Bursting: Neuronal activity alternates, on a **slow timescale**, between a silent phase and an active (bursting) phase of **fast repetitive spikings**

Representative examples of bursting neurons:
chattering neurons in the cortex, thalamocortical relay neurons, thalamic reticular neurons, hippocampal pyramidal neurons, Purkinje cells in the cerebellum, pancreatic β-cells, and respiratory neurons in pre-Botzinger complex



• Complex Brain Network

Network Topology: Complex (Neither Regular Nor Random)

Effect of Network Architecture on Burst and Spike Synchronization

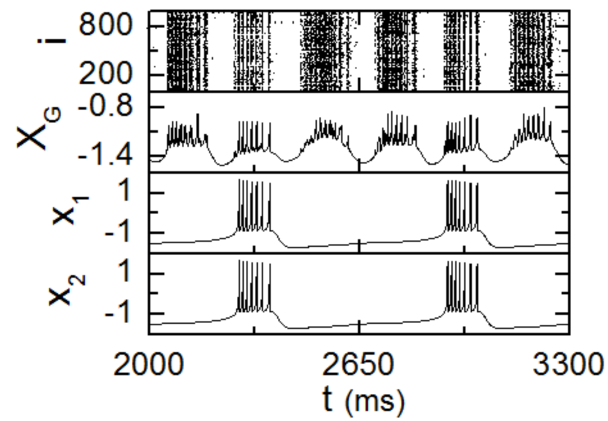
Burst and Spike Synchronizations of Bursting Neurons

- **Synchronization of Bursting Neurons**
  Two Different Synchronization Patterns Due to the **Slow and Fast Time Scales** of Bursting Activity

- **Burst Synchronization**

  Synchrony on the Slow Bursting Timescale

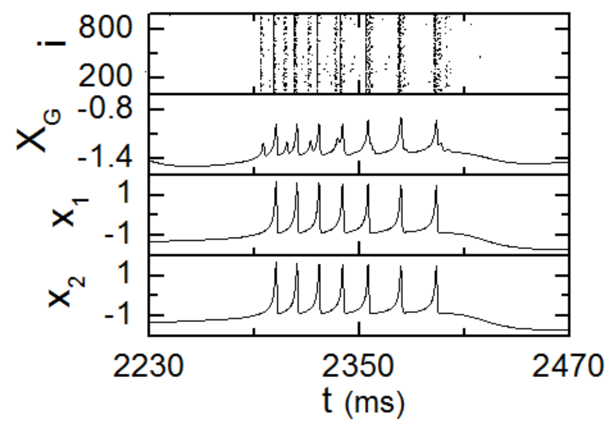
  Temporal coherence between the active phase (bursting) onset or offset times of bursting neurons



- **(Intraburst) Spike Synchronization**

  Synchrony on the Fast Spiking Timescale

  Temporal coherence between intraburst spikes fired by bursting neurons in their respective active phases

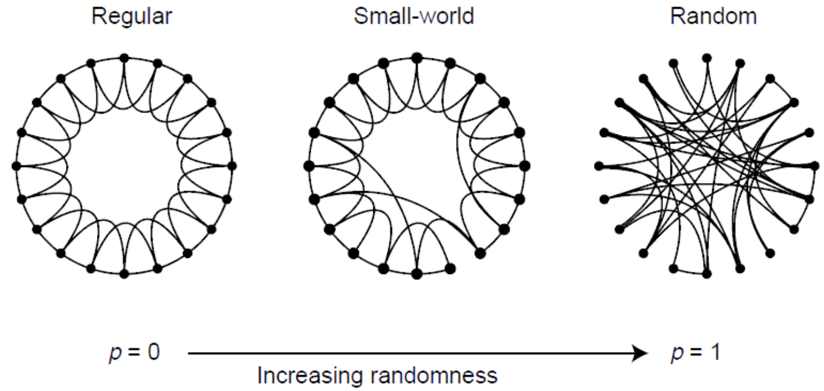


Small-World Network of Inhibitory Bursting Hindmarsh-Rose Neurons

- **Small-World Network of Inhibitory Bursting Hindmarsh-Rose Neurons**

  Start with directed regular ring lattice with N neurons where each neuron is coupled to its first k neighbors.

  Rewire each outward connection at random with probability p such that self-connections and duplicate connections are excluded.



$$\frac{dx_i}{dt} = y_i - ax_i^3 + bx_i^2 - z_i + I_{DC} + D\xi_i - I_{syn,i},$$

$$\frac{dy_i}{dt} = c - dx_i^2 - y_i, \qquad \frac{dz_i}{dt} = r[s(x_i - x_o) - z_i],$$

$$\frac{dg_i}{dt} = \alpha g_\infty(x_i)(1 - g_i) - \beta g_i, \quad i = 1, \ldots, N,$$

$$I_{syn,i} = \frac{J}{N-1}\sum_{j(\neq i)}^{N} w_{i,j} g_j(t)(x_i - X_{syn}), \quad g_\infty(x_i) = 1/[1 + e^{-(x - x_s^*)\delta}].$$

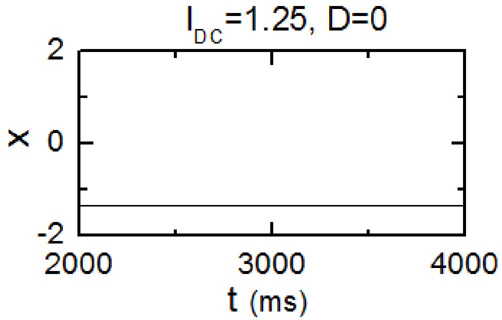
- Parameters in the single HR neuron

  $a = 1,\ b = 3,\ c = 1,\ d = 5,\ r = 0.001,\ s = 4,\ x_o = -1.6$

- Parameters for the synaptic current

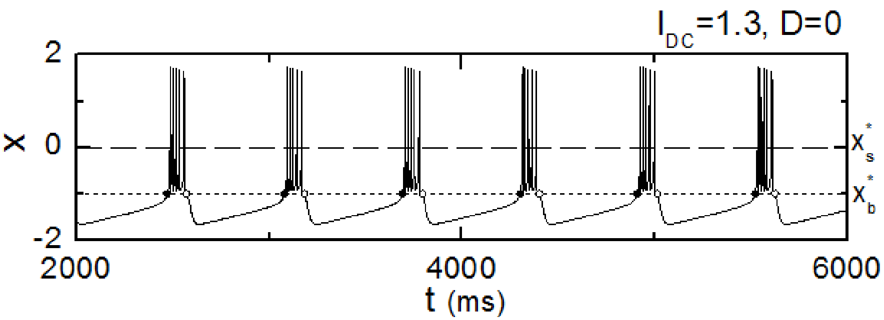
  $X_{syn} = -2,\ x_s^* = 0,\ \delta = 30,\ \alpha = 10\,\mathrm{ms}^{-1},\ \beta = 0.1\,\mathrm{ms}^{-1}$

- **Bursting Activity of the Single HR Neuron**



Transition to a bursting state

$I^*_{DC} \simeq 1.26$



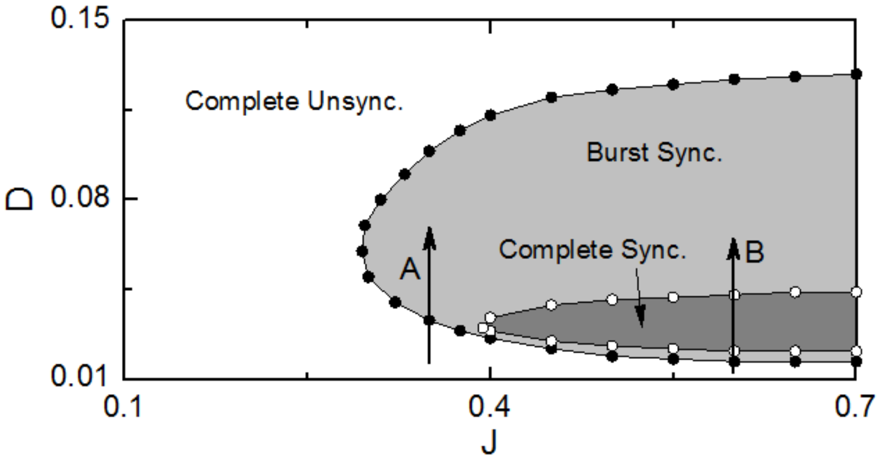
Synchronization in Random Neural Network

- **State Diagram**

Complete Synchronization:
  Including both the burst and spike synchronizations

Bursting Synchronization:
  Without spike synchronization

Complete Unsynchronization



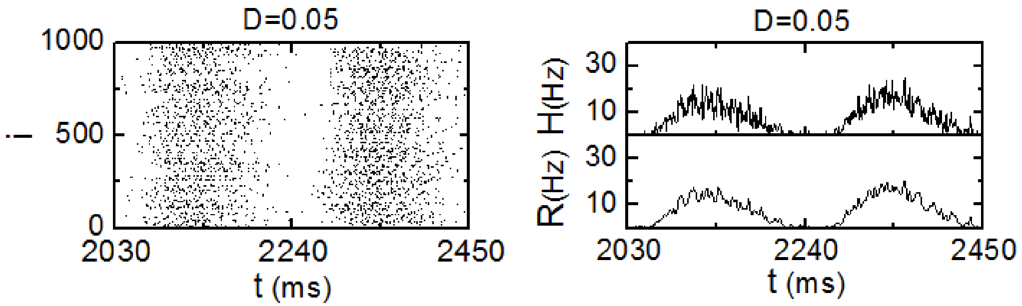
- **Instantaneous Population Firing Rate Kernel Estimate**

Instantaneous population firing rates:

$$R_S(t) \equiv \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{n_i} K_h(t - t_j^{(i)})$$

Gaussian kernel function of band width h:

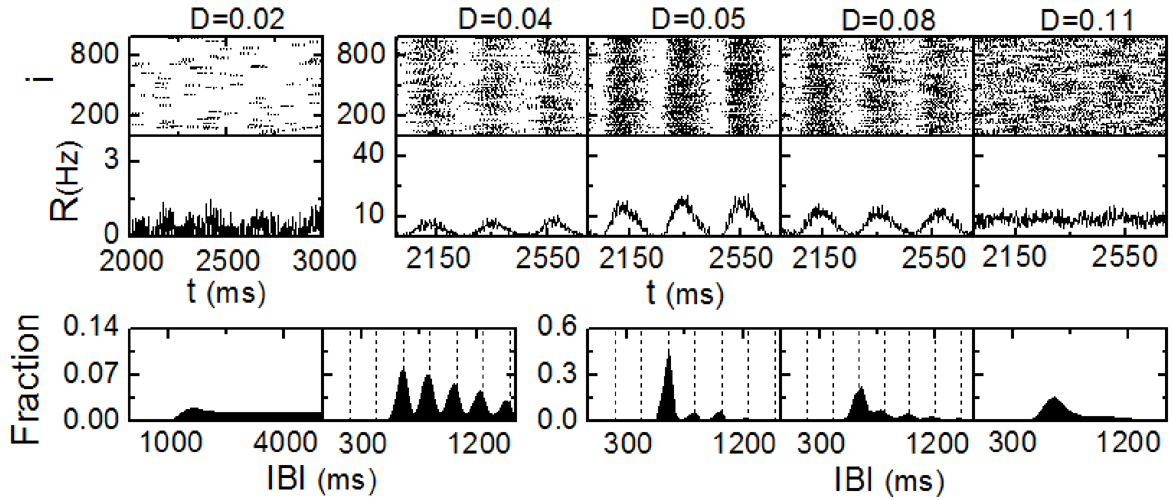
$$K_h(t) = \frac{1}{\sqrt{2\pi}h} e^{-t^2/2h^2}, \qquad -\infty < t < \infty$$



Burst and Spike Synchronization

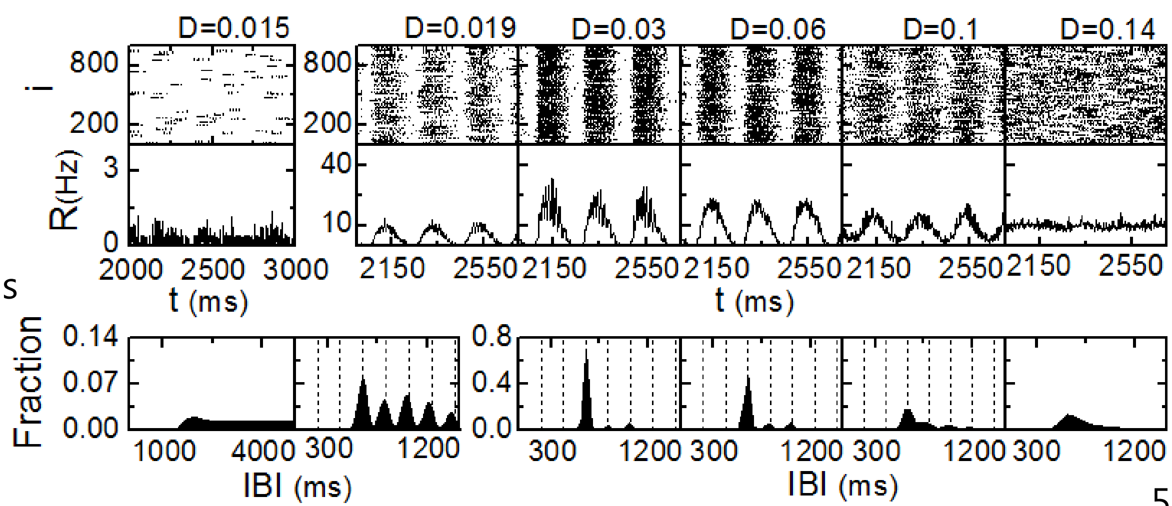
- **Emergence of Burst Synchronization for J=0.35 (Route A)**

Unsynchronization (D=0.02)
↓
Burst Synchronization
  Bursting bands in the raster plot
  Slow oscillation in R(t)
↓
Unsynchronization (D=0.11)



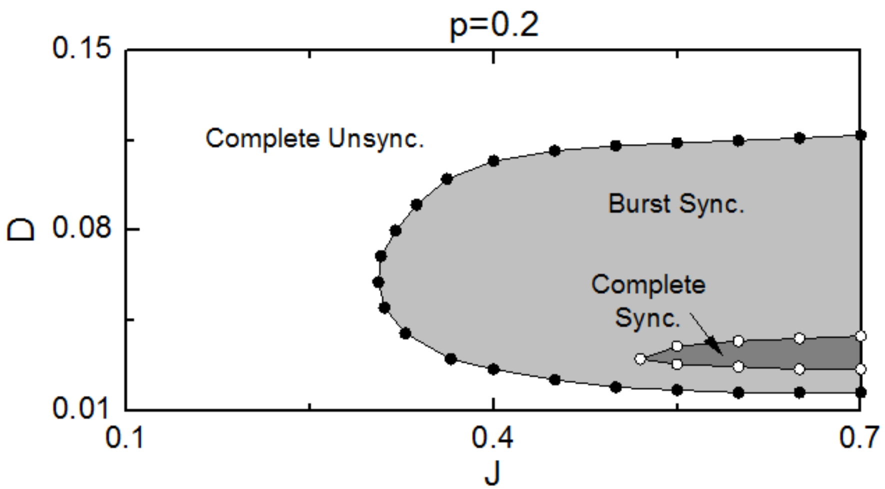
- **Emergence of Burst & Spike Synchronizations for J=0.6 (Route B)**

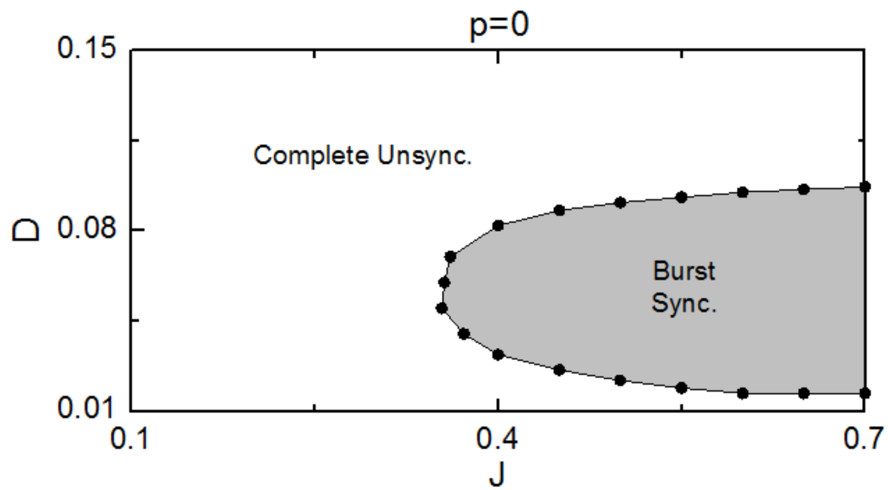
Unsynchronization (D=0.015)
↓
Burst Synchronization
↓
Complete Synchronization (D=0.03)
  Both burst & spike synchronizations
↓
Burst Synchronization
↓
Unsynchronization (D=0.14)



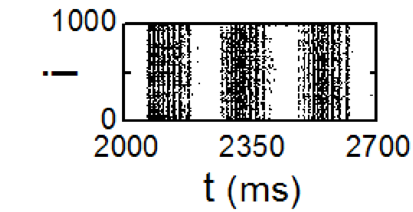
Effect of Network Architecture on the Burst and Spike Synchronization

With decreasing the rewiring probability, the region of burst synchronization decreases slowly, while the region of spike synchronization shrink rapidly.

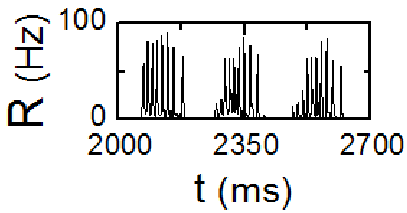




Characterization of Burst and Spike Synchronizations via Separation of the Slow (Bursting) and Fast (Spiking) Time Scales

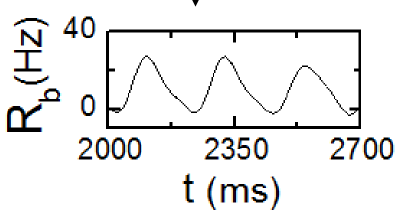


Raster Plot of Neural Spikes:
Population synchronization may be well visualized.
Obtained in experiments



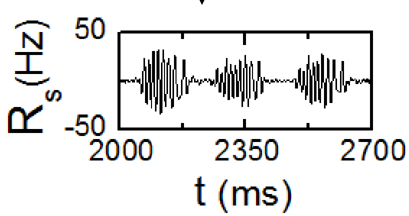
Instantaneous Population Firing Rate (IPFR) R
Describing the population behaviors

Separation of the Slow and Fast Timescale of Bursting Activity via Frequency Filtering



Instantaneous Population Bursting Rate (IPBR) R_b
Describing the Slow Bursting Behavior

Thermodynamic Bursting Order Parameter
Characterization of Bursting Transition



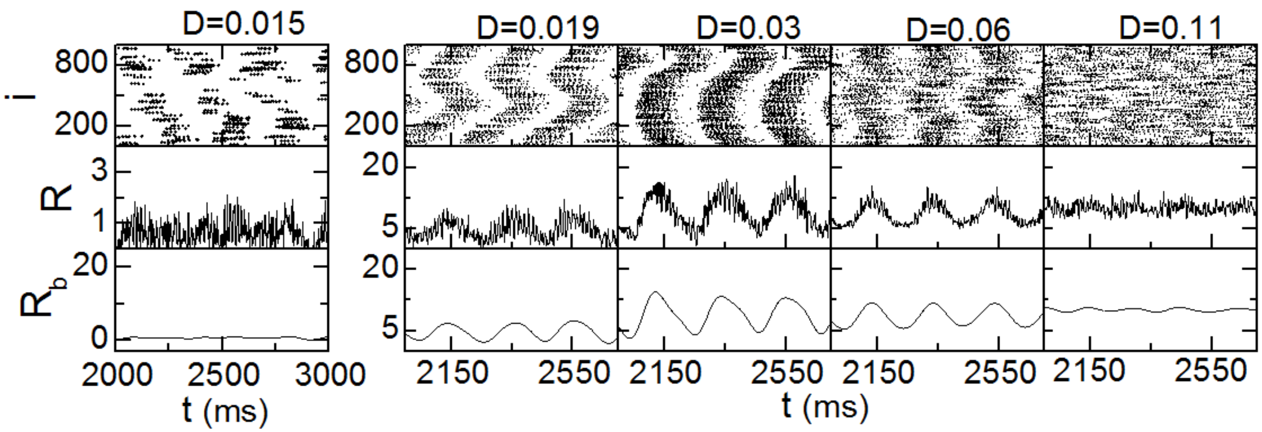
Instantaneous Population Spiking Rate (IPSR) R_s
Describing the Fast Spiking Behavior

Thermodynamic Intraburst Spiking Order Parameter
Characterization of Intraburst Spiking Transition

Statistical-Mechanical Intraburst Spiking Measure
Characterization of Degree of Intraburst Spike Sync.

Characterization of Bursting Transition Based on R_b

- **Investigation of Bursting States via Separation of Slow Timescale**

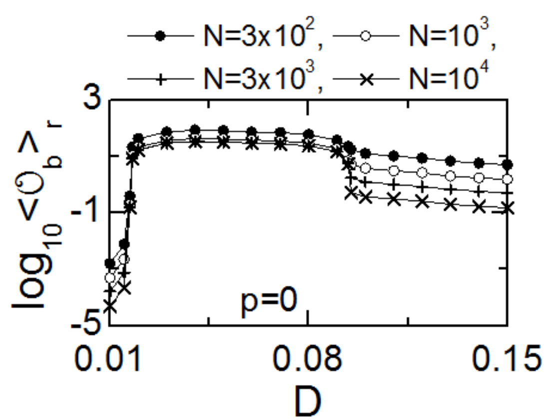


Separation of the slow timescale from IPFR R via low-pass filtering (f_c=10Hz) → IPBR R_b

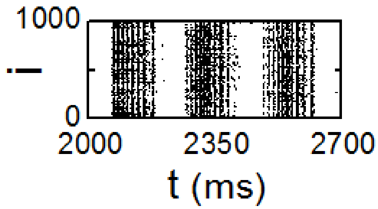
- **Thermodynamic Bursting Order Parameter**

$$\mathcal{O}_b \equiv \overline{(R_b(t) - \overline{R_b(t)})^2}$$

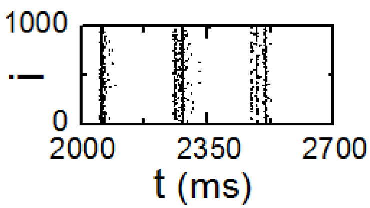
Unsynchronized Bursting State: $N \to \infty$, then $\mathcal{O}_b \to 0$
Synchronized Bursting State:
$N \to \infty$, then $\mathcal{O}_b \to$ Non-zero value

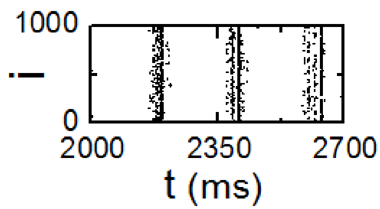


Characterization of Burst Synchronization Based on Bursting Onset and Offset Times



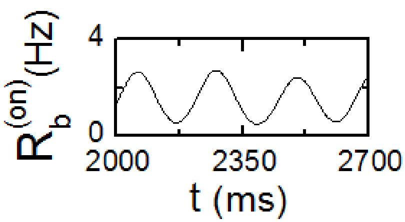
Raster Plot of Neural Spikes

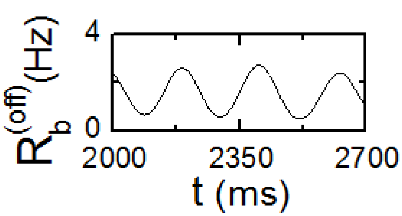




Raster Plots of Active Phase (Bursting) Onset or Offset Times

More direct visualization of bursting behavior





Instantaneous Population Bursting Rates (IPBRs) for Active Phase Onset and Offset Times

Thermodynamic Bursting Order Parameter

Characterization of Bursting Transition

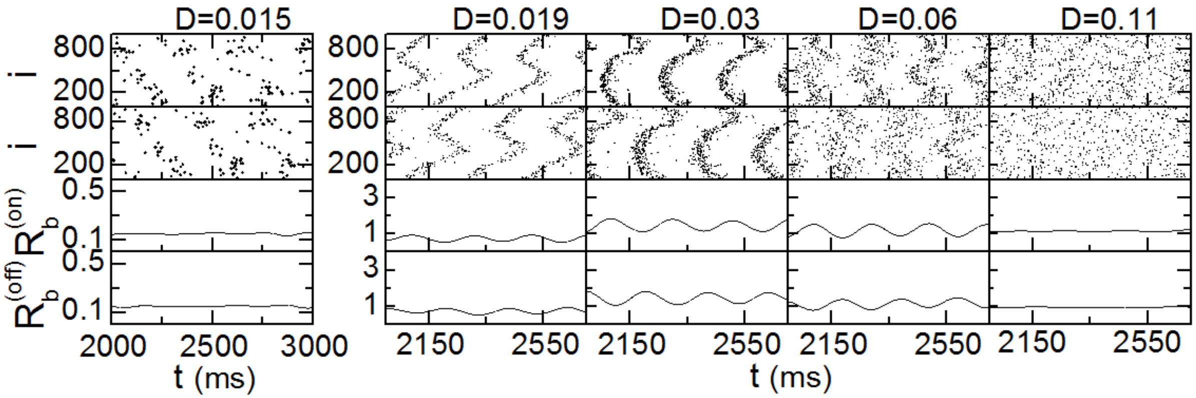
Statistical-Mechanical Bursting Measure

Characterization of Degree of Burst Sync.

Characterization of Bursting Transition Based on $R_b^{(on)}$ and $R_b^{(off)}$

- **Investigation of Bursting States Based on Bursting Onset and Offset Times**

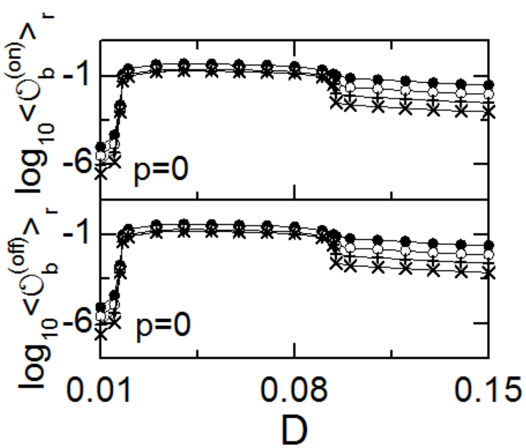
Another Raster plots of bursting onset and offset times and smooth IPBR kernel estimates $R_b^{(on)}$ and $R_b^{(off)}$



- **Thermodynamic Bursting Order Parameters Based on $R_b^{(on)}$ and $R_b^{(off)}$**

$$\mho_b^{(on)} \equiv \overline{(R_b^{(on)}(t) - \overline{R_b^{(on)}(t)})^2}$$

$$\mho_b^{(off)} \equiv \overline{(R_b^{(off)}(t) - \overline{R_b^{(off)}(t)})^2}$$



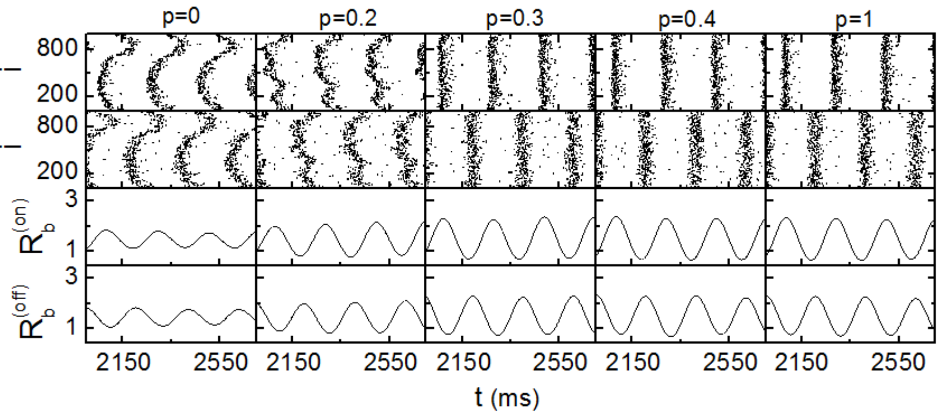
Statistical-Mechanical Bursting Measure Based on $R_b^{(on)}$ and $R_b^{(off)}$

- **Bursting measure** of ith bursting stripe in the raster plot times $M_i^{(b,on)} = O_i^{(b,on)} \cdot P_i^{(b,on)}$

- **Occupation degree** of bursting onset times

$$O_i^{(b,on)} = \frac{N_i^{(b)}}{N}$$

$N_i^{(b)}$: No. of bursting HR neurons in the ith bursting stripe



- **Pacing degree** of bursting onset times: contribution of bursting onset times to the macroscopic IPBR $R_b^{(on)}$

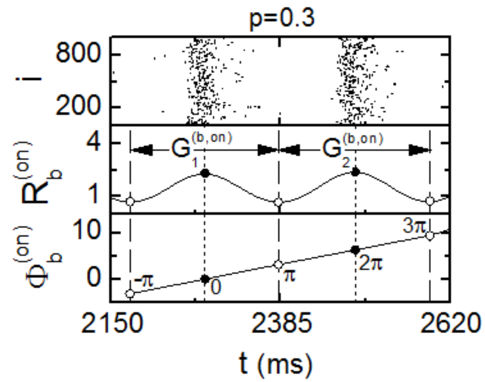
$$P_i^{(b,on)} = \frac{1}{B_i^{(on)}} \sum_{k=1}^{B_i^{(on)}} \cos \Phi_k$$

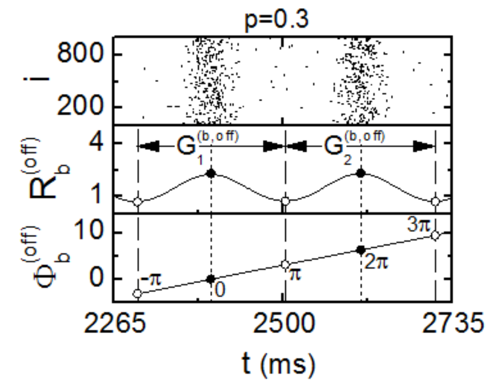
Φ_k: Global bursting phase based on IPBR $R_b^{(on)}$

$B_i^{(on)}$: Total No. of microscopic bursting onset times in the ith bursting stripe

- **Statistical-Mechanical Bursting Measure** Based on IPBR $R_b^{(on)}$

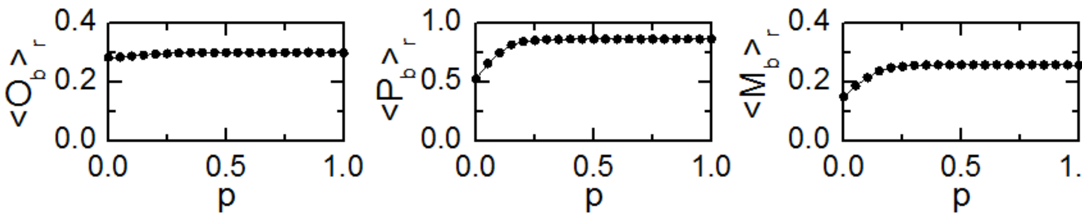
$$M_b^{(on)} = \frac{1}{N_b} \sum_{i=1}^{N_b} M_i^{(b,on)}$$





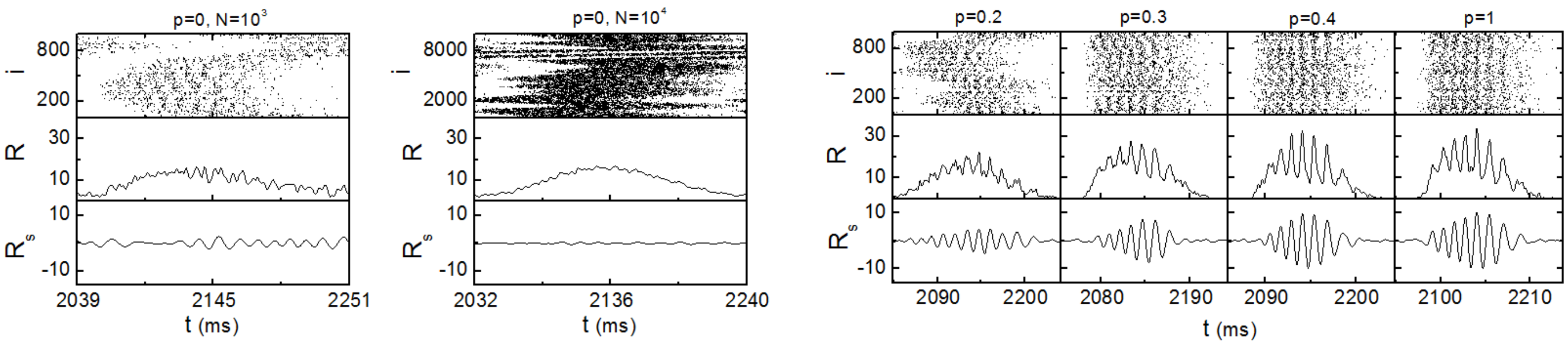
- **Statistical-Mechanical Bursting Measure**

$$M_b = [M_b^{(on)} + M_b^{(off)}]/2$$



Characterization of Intraburst Spiking Transition Based on R_s

- **Investigation of Intraburst Spiking States via Separation of Fast Time Scale**
  - IPSR R_s via band-pass filtering [lower and higher cut-off frequencies of 30Hz (high-pass filter) and 90 Hz (low-pass filter)]

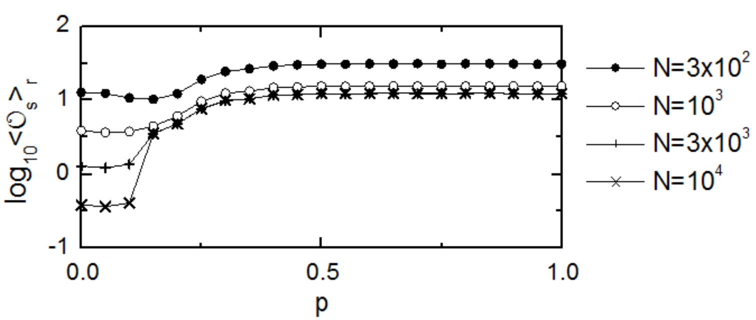


- **Thermodynamic Intraburst Spiking Order Parameter**

Mean square deviation of R_s in the ith global bursting cycle: $\mathcal{O}_s^{(i)} \equiv \overline{(R_s(t) - \overline{R_s(t)})^2}$

Thermodynamic spiking order parameter:

$$\mathcal{O}_s = \frac{1}{N_b} \sum_{i=1}^{N_b} \mathcal{O}_s^{(i)}$$



Unsynchronized Spiking State: $N \to \infty$, then $\mathcal{O}_s \to 0$
Synchronized Spiking State:
$N \to \infty$, then $\mathcal{O}_s \to$ Non-zero value

Intraburst Spiking Measure Based on IPSR R_s

- **Spiking measure** of jth global spiking cycle in the ith bursting cycle $\quad M_{i,j}^{(s)} = O_{i,j}^{(s)} \cdot P_{i,j}^{(s)}$

- **Occupation degree** of spiking times

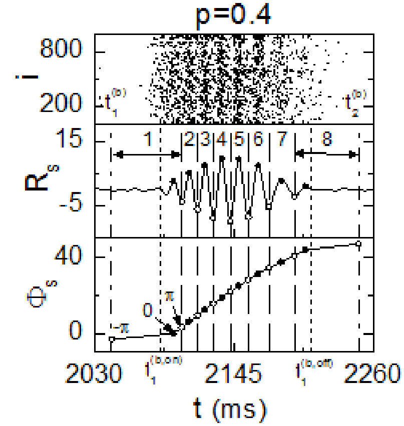
$$O_{i,j}^{(s)} = \frac{N_{i,j}^{(s)}}{N}$$

$N_{i,j}^{(s)}$: No. of spiking HR neurons in the jth spiking cycle and the ith bursting cycle

- **Pacing degree** of spiking times: Contribution of spiking times to the macroscopic IPSR R_s

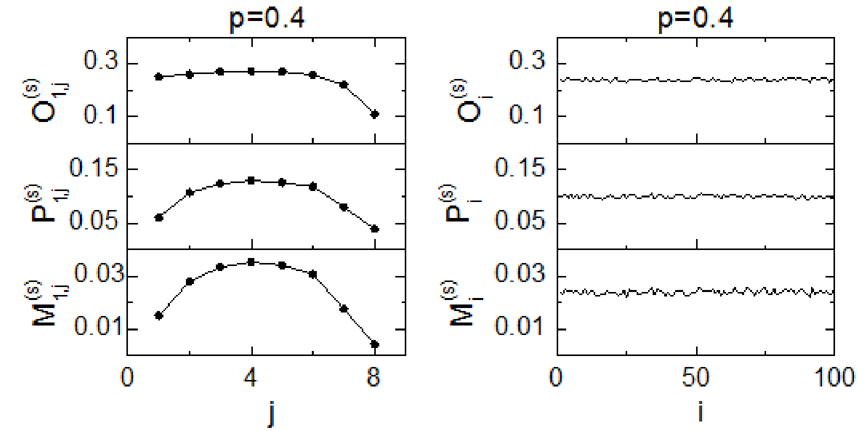
$$P_{i,j}^{(s)} = \frac{1}{N_{i,j}^{(s)}} \sum_{k=1}^{N_{i,j}^{(s)}} \cos \Phi_k$$

$N_{i,j}^{(s)}$: Total No. of microscopic spiking times



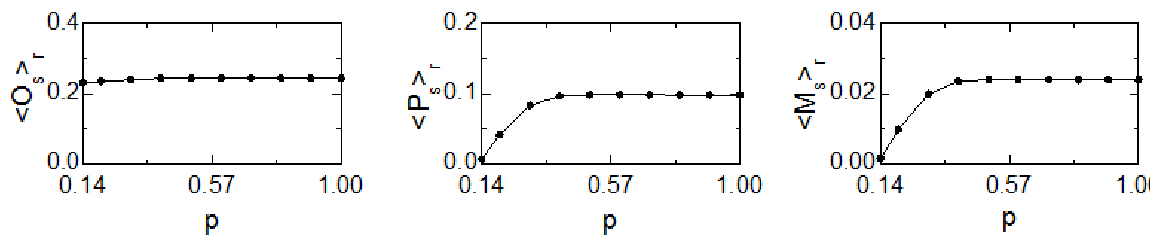
- **Statistical-Mechanical Spiking Measure** Based on IPSR R_s

$$M_i^{(s)} = \frac{1}{N_s} \sum_{j=1}^{N_s} M_{i,j}^{(s)}$$



- **Statistical-Mechanical Spiking Measure**

$$M_s = \frac{1}{N_b} \sum_{i=1}^{N_b} M_i^{(s)}$$



Summary

- **Investigation of The Effect of Network Architecture on the Burst and Spike Synchronization**

  Occurrence Burst and Spike Synchronization in Random Network

  Investigation of Burst and Spike Synchronization with Decreasing the Rewiring Probability

  Burst synchronization decreases slowly, while spike synchronization shrink rapidly.

  Characterization of Burst and Spike Synchronization by Using the Realistic Thermodynamic Bursting and Spiking Order Parameter Based on the IPBR and IPSR Kernel Estimates through Frequency Filtering.

  Characterization of Degree of Burst and Spike Synchronization by Using the Realistic Statistical-Mechanical Bursting and Spiking Measure Based on the IPBR and IPSR Kernel Estimates through Frequency Filtering.