Effect of Inter-Modular Connection on Fast Sparse Synchronization in Clustered Small-World Networks[1]

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BACKGROUND

- **Fast Sparsely Synchronized Brain Rhythms**
  - Population level: Fast synchronous oscillations [e.g. gamma rhythm (30~100Hz) and sharp-wave ripple (100~200Hz)]
  - Cellular level: Stochastic and intermittent discharges
  - Associated with diverse cognitive functions [e.g., sensory perception, feature integration, selective attention, and memory formation and consolidation]
  - Previous works of Brunel et al.[2]: Developed a framework appropriate for fast sparse synchronization in global and random networks. But, realistic brain networks: neither regular nor random
- **Modular Architecture of Real Brain**
  - Modular structure of brain: The mammalian brain (e.g., cat and macaque) anatomical network and human brain functional network has been revealed to have a modular structure composed of sparsely linked clustered with spatial localization.
  - Complex topology in modules: Revealing complex topology such as small-worldness and scale-freeness which are neither regular nor random.
  - Our neuronal model: Clustered Small-World Network (SWN)

SPECIFIC AIMS

- To Study Emergence of Sparsely Synchronized Brain Rhythms in Clustered SWNs
- To Investigate The Effect of Inter-Modular Connection on Sparse Synchronization by Varying The Inter-Modular Coupling Strength J_{inter} and The Average Number of Links per Neuron $M_{syn}^{(inter)}$

METHODS

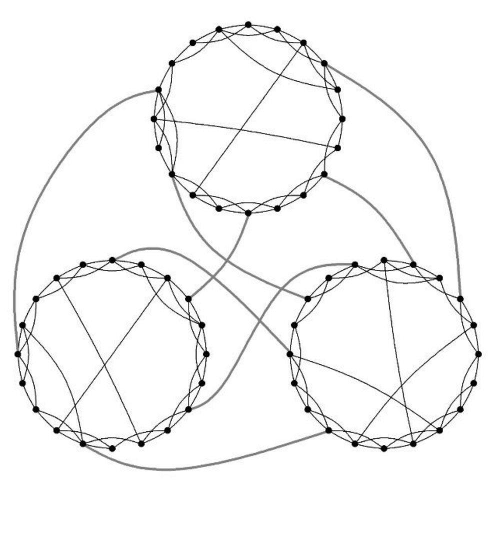
- **Clustered Networks with 3 Small-World (SW) Sub-Networks**

Each Cluster: Small-World Sub-Network Composed of L Inhibitory Fast Spiking Izhikevich Interneurons[3]

Watts-Strogatz SWN Model[4]: Interpolating between the regular lattice with high clustering and the random graph with short path length by varying the rewiring probability $p_{rewiring}$ from local to long-range connections; $p_{rewiring}$=0 (regular lattice) & $p_{rewiring}$=1 (random graph)

Intra-modular connection: Small-World Links with predominantly local connections and rare long-range connections

Inter-modular connection: Sparse Random Links



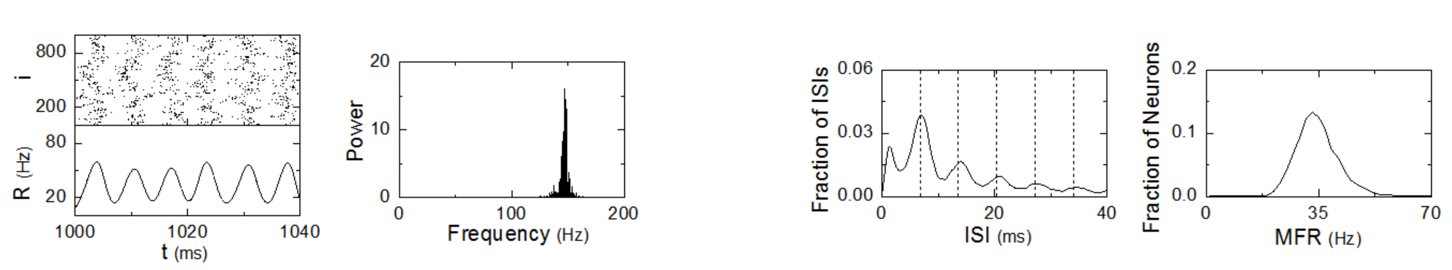
RESULTS

1. Fast Sparsely Synchronized Brain Rhythms in SW Sub-Networks

$(I_{DC}=1500,\ M_{syn}^{(intra)}=50,\ J_{intra}=1400,\ D=500,\ p_{rewiring}=0.25)$

- **Intra-Modular Dynamics**

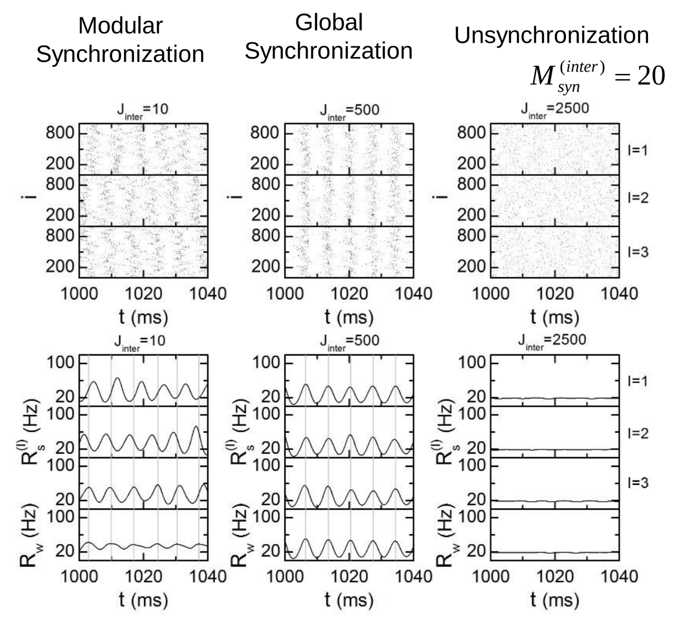
Fast Sparsely Synchronized State with the population frequency f_p=147 Hz
Stochastic and Intermittent Individual Spike Discharge with mean firing rate f_i=33 Hz.



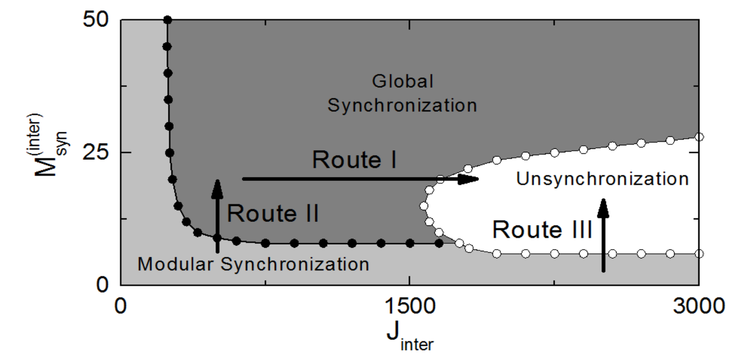
RESULTS (Continued)

2. Modular and Global Synchronization in Clustered SWN

- **Instantaneous Population Spike Rate (IPSR)**
  - **Instantaneous sub-population spike rate (ISPSR) for the *I*th sub-network** $R_s^{(I)}(t)$: Obtained via convolution (or blurring) of each spike in the raster plot with the Gaussian kernel function
  - **Instantaneous whole-population spike rate (IWPSR) for the whole network** $R_w(t)$ (corresponding to the average of all ISPSRs)



- **State Diagram in the $J_{inter}-M_{syn}^{(inter)}$ Plane**

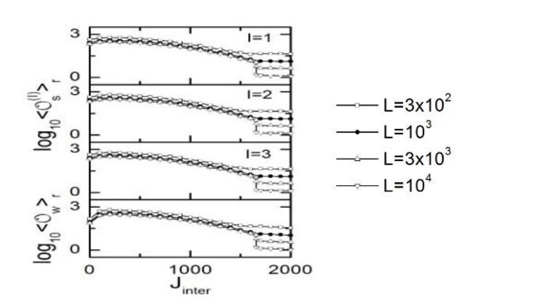


Modular Synchronization: Mismatching between intra-modular dynamics of sub-networks
Global Synchronization: Perfect Matching between intra-modular dynamics of sub-networks

3. Synchronization-Unsynchronization Transition (Route I with $M_{syn}^{(inter)}=20$)

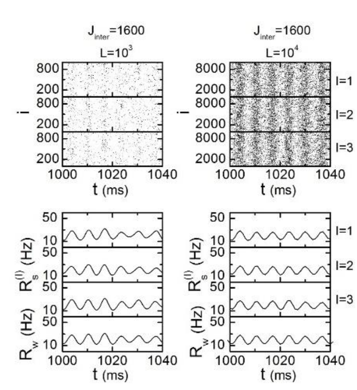
- **Realistic Thermodynamic Order Parameter**
  - **Sub-population order parameter $\mathcal{O}_s^{(I)}$ for the *I*th sub-network**: Representing the time-averaged fluctuation of the ISPSR $R_s^{(I)}(t)$
  - **Whole-population order parameter $\mathcal{O}_w$ for the whole network:** Representing the time-averaged fluctuation of the IWPSR $R_w(t)$

For the synchronized (unsynchronized) state, $\mathcal{O}_s^{(I)}$ and $\mathcal{O}_w$ approach non-zero (zero) limit values in the thermodynamic limit of $L\to\infty$.

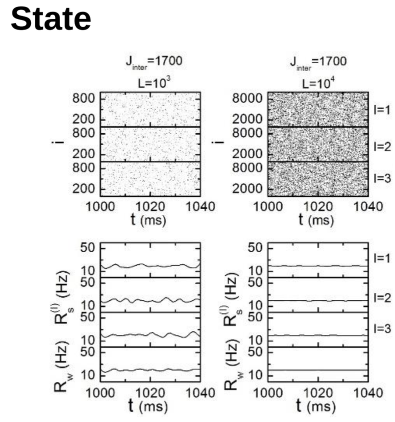


When passing the threshold $J_{inter,h}^*$ (~1657), a transition to unsynchronization occurs.

- **Synchronized State**
  Sparse stripes are formed in the raster plot
  $R_s^{(I)}(t)$ and $R_w(t)$ show regular oscillations



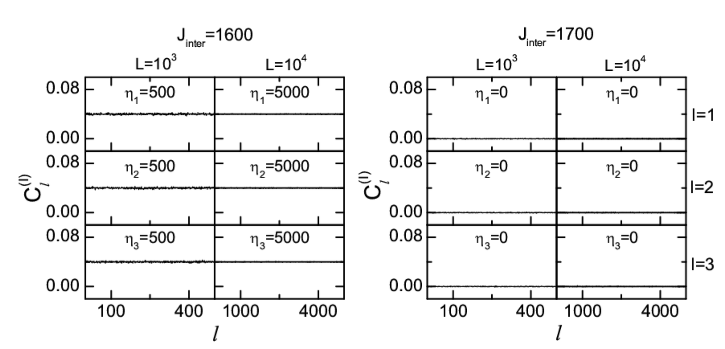
- **Unsynchronized State**
  Sparse spikes are scattered in the raster plot
  $R_s^{(I)}(t)$ and $R_w(t)$ becomes nearly stationary



4. Characterization of Synchronization and Unsynchronization in Terms of the Spatial Cross-Correlations (Route I with $M_{syn}^{(inter)}=20$)

- **Spatial Cross-Correlations**
  - Instantaneous individual spike rate (IISR) $r_i^{(I)}(t)$ of the *i*th neuron in the *I*th sub-network: Obtained via convolution of each spike of the *i*th neuron in the *I*th sub-network with the Gaussian kernel function
  - Normalized temporal cross-correlation function $C_{i,j}^{(I)}(\tau)$ between in the IISRs of the (i,j) neuronal pair in the *I*th sub-network
  - Spatial cross-correlation function $C_l^{(I)}$ between neuronal pairs separated by a spatial distance *l* in the *I*th sub-network: Average of all the temporal cross-correlations between IISRs of the $(i, i+l)$ neuronal pairs in the *I*th sub-network at the zero-time lag

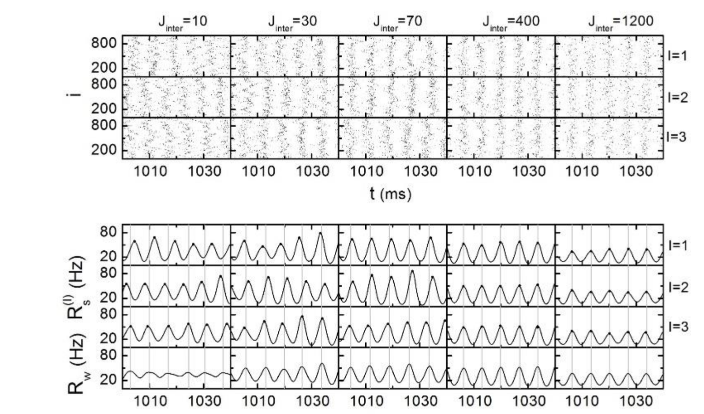
For a synchronized state:
$C_l^{(I)}$: nearly non-zero constant for whole range of *l*. Correlation length = *L*/2
For an unsynchronized state:
$C_l^{(I)}$: nearly zero for whole range of *l*. Correlation length = 0



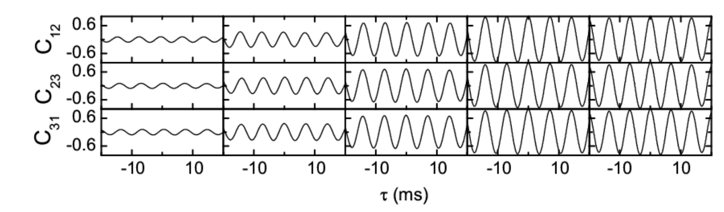
RESULTS (Continued)

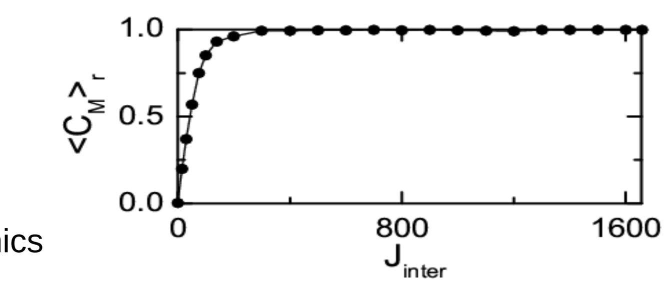
5. Modular-Global Synchronization Transition (Route I with $M_{syn}^{(inter)}=20$)

- **Modular and Global Synchronization**



$C_{IJ}(\tau)$: Normalized temporal cross-correlation function between the ISPSRs of the *I*th and *J*th sub-networks





- **Cross-Correlation Modularity Measure C_M**
  Representing the matching degree between the intra-dynamics of sub-networks & Obtained via average of the temporal cross-correlations between all the sub-population pairs at the zero-time lag

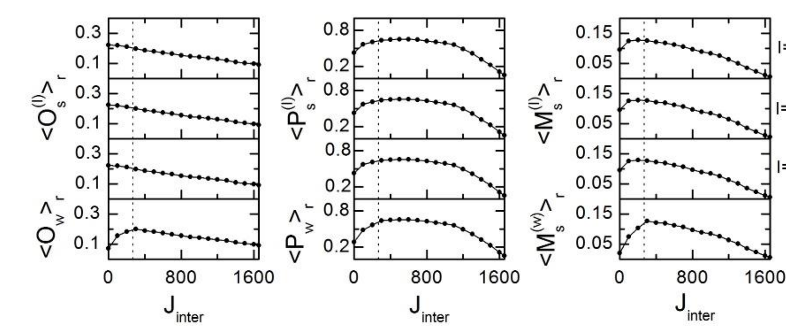
$J_{inter} < J_{inter,l}^*$ (~268) : Modular Sync. : $0<\langle C_M\rangle_r<1$
$J_{inter,l}^* < J_{inter} < J_{inter,h}^*$: Global Sync. : $\langle C_M\rangle_r \simeq 1$

6. Characterization of Synchronization Degree (Route I with $M_{syn}^{(inter)}=20$)

- **Realistic Statistical-Mechanical Spiking Measure[5]**
  - **Sub- & Whole-Population Spiking Measures** $M_s^{(I)}$ and $M_s^{(w)}$: Given by the product of the sub- & whole-population occupation and the sub- & whole-population pacing degrees of spikes in the raster plot.
  - **Sub- & Whole-Population Occupation degrees** $O_s^{(I)}$ and O_w: representing the density of stripes in the raster plot
  - **Sub- & Whole-Population Pacing degrees** $P_s^{(I)}$ and P_w : representing the smearing of stripes in the raster plot (average contribution of all microscopic spikes to the instantaneous population spike rate)

For the case of modular synchronization, O_w and P_w are less than $O_s^{(I)}$ and $P_s^{(I)}$ because of mismatching between the intra-modular dynamics of sub-networks. → $M_s^{(w)}$ is less than $M_s^{(I)}$
For the case of global synchronization, $O_w=O_s^{(I)}$, $P_w=P_s^{(I)}$, and $M_s^{(w)}=M_s^{(I)}$.

With increasing J_{inter} ,
$O_s^{(I)}$ decreases monotonically because of increase in inhibition, and O_w increases and approaches $O_s^{(I)}$ due to decrease in the mismatching degree between the intra-modular dynamics of sub-networks.
$P_s^{(I)}$ and P_w exhibits the bell-shaped curve.
→ Spiking measures $M_s^{(I)}$ and $M_s^{(w)}$ exhibit the bell-shaped curve.



DISCUSSIONS

- **Investigated The Effect of Inter-Modular Connections on Emergence of Sparsely Synchronized Modular and Global Brain Rhythms by Varying J_{inter} (inter-modular coupling strength) and $M_{syn}^{(inter)}$ (average number of links per interneuron)**
- **Roles of J_{inter} and $M_{syn}^{(inter)}$**
  Dual Roles of Inter-Modular Coupling Strength (Route I):
  For small J_{inter} → Constructive role to "favor" the pacing between sparse spikes
  For large J_{inter} → Destructive role to "spoil" the pacing between sparse spikes
  Role of Number of Inter-Modular Connections (Routes II & III):
  Constructive role just to "favor" global communication between sub-networks (i.e., to increase the pacing degree between sparse spikes)
- **Implications for The Role of The Brain Plasticity**
  Since changes in the coupling strengths and the synaptic connections are closely interwoven with the brain plasticity, we expect that our results on the inter-modular connection effect in modular networks could have Implications for the role of the brain plasticity in some functional behaviors associated with population synchronization

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[1] Kim S.-Y. & Lim W. (2015) Effect of inter-modular connection on fast sparse synchronization in clustered small-world neural networks. Submitted for publication in the Phys. Rev. E (arXiv:1507.03311 [q-bio.NC])
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