Thermodynamic and Statistical-Mechanical Measures for Synchronization of Bursting Neurons

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Introduction

- **Burstings with the Slow and Fast Timescales**
  Bursting: Neuronal activity alternates, on a slow timescale, between a silent phase and an active (bursting) phase of fast repetitive spikings
  Representative examples of bursting neurons: chattering neurons in the cortex, thalamocortical relay neurons, thalamic reticular neurons, hippocampal pyramidal neurons, Purkinje cells in the cerebellum, pancreatic β-cells, and respiratory neurons in pre-Botzinger complex
- **Synchronization of Bursting Neurons**
  Two Different Synchronization Patterns Due to the Slow and Fast Timescales of Bursting Activity
- **Burst Synchronization: Synchrony on the Slow Bursting Timescale**
  Temporal coherence between the active phase (bursting) onset or offset times of bursting neurons
- **Spike Synchronization: Synchrony on the Fast Spiking Timescale**
  Temporal coherence between intraburst spikes fired by bursting neurons in their respective active phases

Inhibitory Population of Bursting Neurons

$$\frac{dx_i}{dt} = y_i - a x_i^3 + b x_i^2 - z_i + I_{DC} + D\xi_i - I_{syn,i},$$
$$\frac{dy_i}{dt} = c - d x_i^2 - y_i, \quad \frac{dz_i}{dt} = r[s(x_i - x_o) - z_i],$$
$$\frac{dg_i}{dt} = \alpha g_\infty(x_i)(1-g_i) - \beta g_i, \quad i = 1, \ldots, N,$$
$$I_{syn,i} = \frac{J}{N-1}\sum_{j(\neq i)}^{N} g_j(t)(x_i - X_{syn}), \quad g_\infty(x_i) = 1/[1+e^{-(x_i - x_s^*)/\delta}].$$

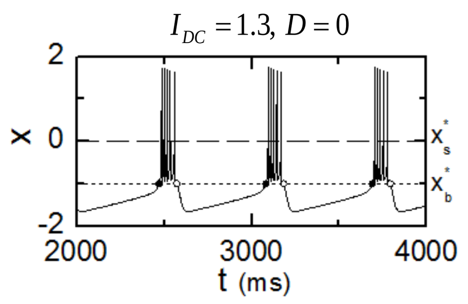
Parameters in the single Hindmarsh-Rose (HR) neuron

$a=1,\ b=3,\ c=1,\ d=5,\ r=0.001,\ s=4,\ x_o=-1.6$

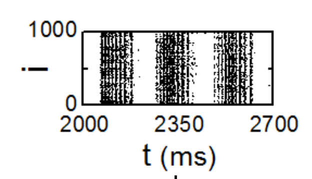
Parameters for the synaptic current

$X_{syn}=-2,\ x_s^*=0,\ \delta=30,\ \alpha=10\,\mathrm{ms}^{-1},\ \beta=0.1\,\mathrm{ms}^{-1}$

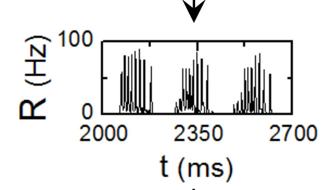
- **Bursting Activity of the Single HR Neuron**
  Transition to a bursting state occurs for $I_{DC}^* \simeq 1.26$
  Dotted horizontal line ($x_b^* = -1$): bursting threshold
  (solid & open circles: bursting onset & offset times)
  Dashed horizontal line ($x_s^* = 0$): spiking threshold



Characterization of Burst and Spike Synchronizations via Separation of the Slow (Bursting) and Fast (Spiking) Timescales

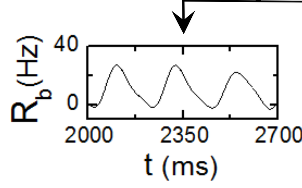


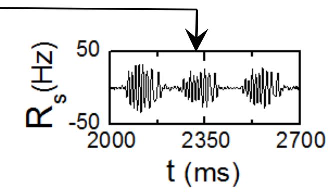
Raster Plot of Neural Spikes:
Population synchronization may be well visualized.
Obtained in experiments



Instantaneous Population Firing Rate (IPFR) R
Describing the population behaviors

Separation of the Slow and Fast Timescale of Bursting Activity via Frequency Filtering

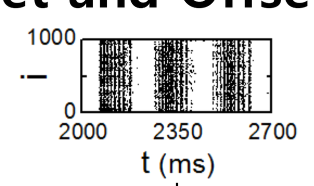




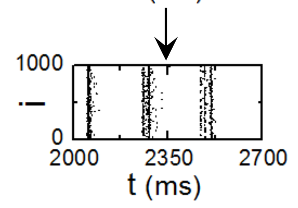
Instantaneous Population Bursting Rate (IPBR) R_b
Describing the Slow Bursting Behavior

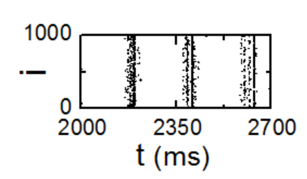
Instantaneous Population Spiking Rate (IPSR) R_s
Describing the Fast Spiking Behavior

Characterization of Burst Synchronization Based on Bursting Onset and Offset Times

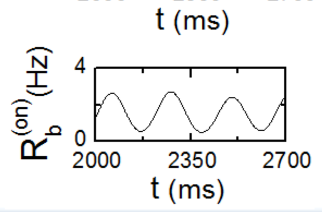


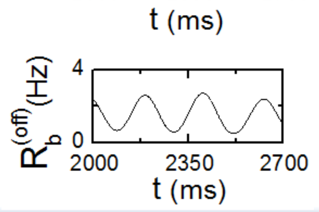
Raster Plot of Neural Spikes





Raster Plots of Active Phase (Bursting) Onset or Offset Times
More direct visualization of bursting behavior

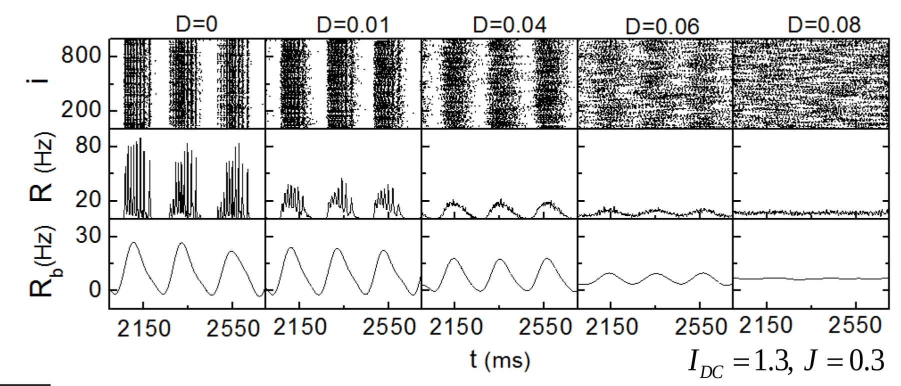




Instantaneous Population Bursting Rates (IPBRs) for Active Phase Onset Time $R_b^{(on)}$ **and Offset Time** $R_b^{(off)}$

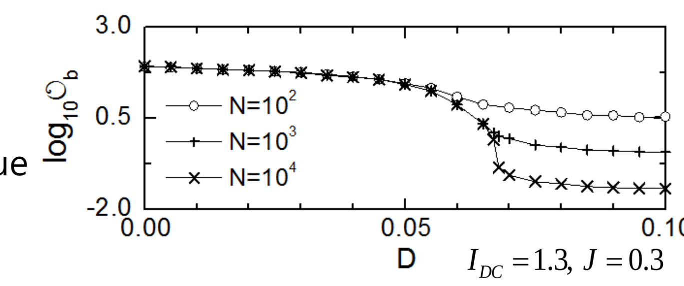
Determination of Threshold for Bursting Transition via Thermodynamics Bursting Order Parameter

- **Thermodynamic Bursting Order Parameter Based on** R_b
  IPBR R_b via low-pass filtering (f_c=10Hz)
  As D is increased, bursting bands in the raster plots becomes smeared and overlap.
  → Amplitude of R_b decreases.

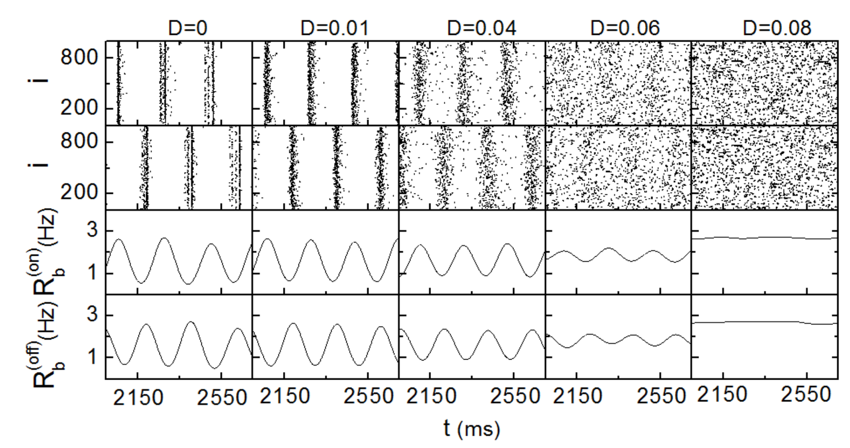


- Thermodynamic Bursting Order Parameter
  Mean square deviation of R_b: $\mathcal{O}_b \equiv \overline{(R_b(t) - \overline{R_b(t)})^2}$

Unsynchronized Bursting State: $N\to\infty$, then $\mathcal{O}_b \to 0$
Synchronized Bursting State: $N\to\infty$, then $\mathcal{O}_b \to$ Non-zero value
Bursting Transition occurs for $D_b^* (\simeq 0.068)$



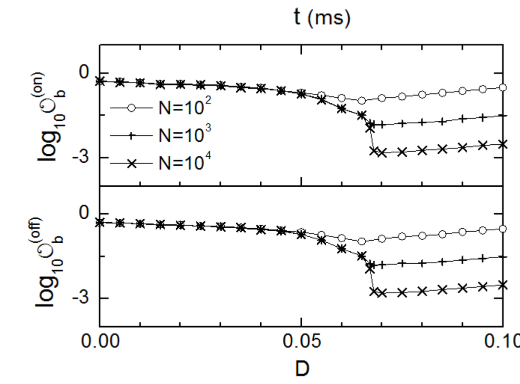
- **Thermodynamic Bursting Order Parameters Based on** $R_b^{(on)}$ **and** $R_b^{(off)}$
  Another Raster plots of bursting onset and offset times and smooth IPBR kernel estimates $R_b^{(on)}$ and $R_b^{(off)}$
  As D is increased, bursting stripes in the raster plots becomes smeared and overlap.
  → Amplitude of both $R_b^{(on)}$ and $R_b^{(off)}$ decreases.



- Thermodynamic Bursting Order Parameter
  Mean square deviations of $R_b^{(on)}$ and $R_b^{(off)}$:

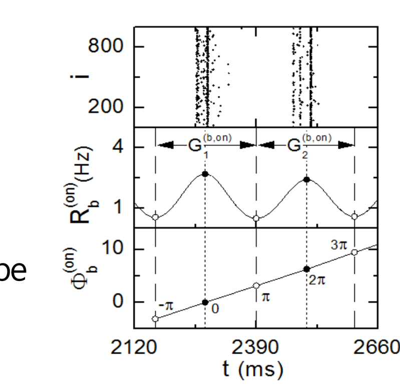
$$\mathcal{O}_b^{(on)} \equiv \overline{(R_b^{(on)}(t) - \overline{R_b^{(on)}(t)})^2}$$
$$\mathcal{O}_b^{(off)} \equiv \overline{(R_b^{(off)}(t) - \overline{R_b^{(off)}(t)})^2}$$

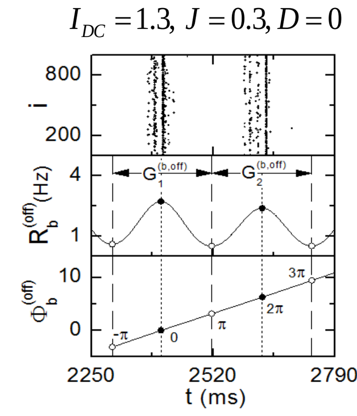
Bursting Transition occurs for $D_b^* (\simeq 0.068)$

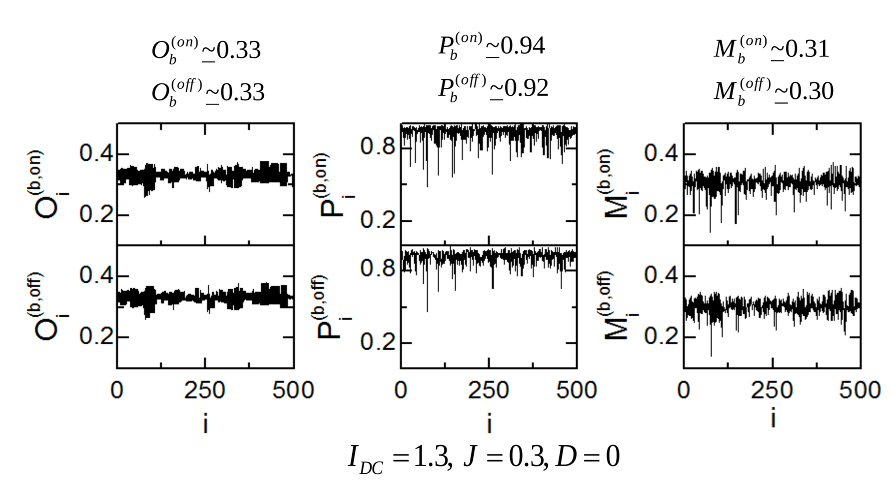


Measurement of Degree of Bursting Synchronization via Statistical-Mechanical Bursting Measure

- **Bursting measure** of ith bursting stripe in the raster plots of bursting onset times
  $M_i^{(b,on)} = O_i^{(b,on)} \cdot P_i^{(b,on)}$
- **Occupation degree** of bursting onset times:
  $O_i^{(b,on)} = \frac{N_i^{(b,on)}}{N}$ $N_i^{(b,on)}$: No. of HR neurons which fire bursting in the ith bursting onset stripe
- **Pacing degree** of bursting onset times:
  Contribution of bursting onset times to the macroscopic IPBR $R_b^{(on)}$
  $$P_i^{(b,on)} = \frac{1}{B_i^{(on)}}\sum_{k=1}^{B_i^{(on)}} \cos\Phi_k$$
  Φ_k : Global bursting phase based on IPBR $R_b^{(on)}$
  $B_i^{(on)}$: Total No. of microscopic bursting onset times in the ith bursting stripe

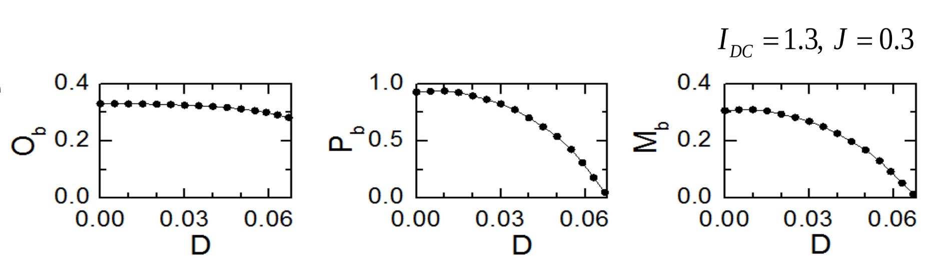






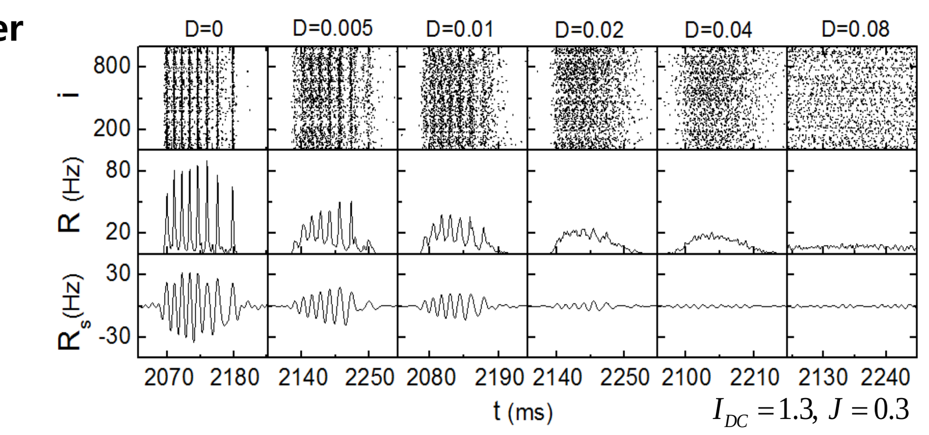
- **Statistical-Mechanical Bursting Measure** based on IPBR $R_b^{(on)}$
  $$M_b^{(on)} = \frac{1}{N_b}\sum_{i=1}^{N_b} M_i^{(b,on)}$$
- **Statistical-Mechanical Bursting Measure**
  $$M_b = [M_b^{(on)} + M_b^{(off)}]/2$$

With increasing D, O_b decreases very slowly, while P_b and M_b decrease very rapidly.



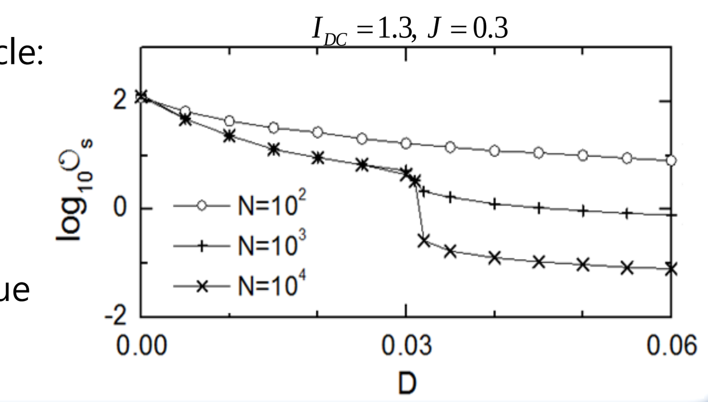
Determination of Threshold for Intraburst Spiking Transition via Thermodynamic Intraburst Spiking Order Parameter

- **Thermodynamic Intraburst Spiking Order Parameter**
  IPSR R_s via band-pass filtering
  [lower and higher cut-off frequencies of 30Hz (high-pass filter) and 90 Hz (low-pass filter)]
  As D is increased, stripes in the burst band becomes smeared and overlap.
  → Amplitude of R_s decreases.



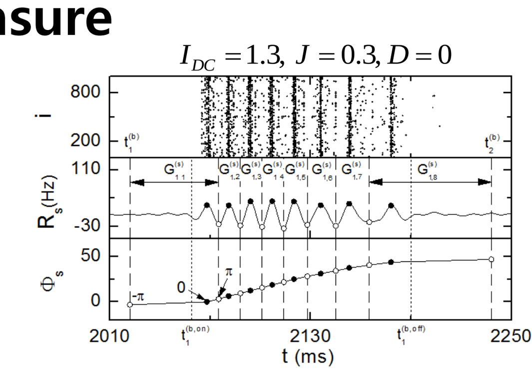
Mean square deviation of R_s in the ith global bursting cycle:
$$\mathcal{O}_s^{(i)} \equiv \overline{(R_s(t) - \overline{R_s(t)})^2}$$
Thermodynamic spiking order parameter: $\mathcal{O}_s = \frac{1}{N_b}\sum_{i=1}^{N_b}\mathcal{O}_s^{(i)}$

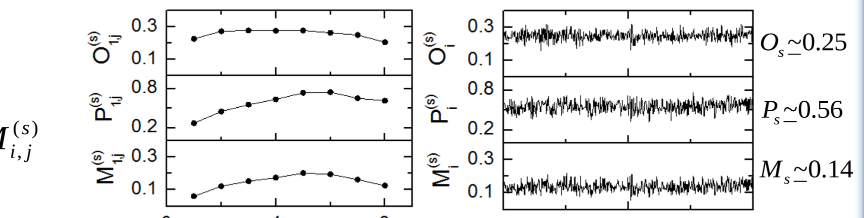
Unsynchronized Spiking State: $N\to\infty$, then $\mathcal{O}_s \to 0$
Synchronized Spiking State: $N\to\infty$, then $\mathcal{O}_s \to$ Non-zero value
Intraburst Spiking Transition occurs for $D_s^* (\simeq 0.032)$

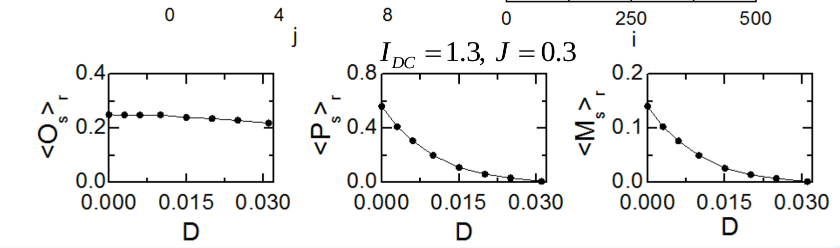


Measurement of Degree of Intraburst Spike Synchronization via Statistical-Mechanical Intraburst Spiking Measure

- **Spiking measure** of jth global spiking cycle in the ith bursting cycle $M_{i,j}^{(s)} = O_{i,j}^{(s)} \cdot P_{i,j}^{(s)}$
- **Occupation degree** of spiking times: $O_{i,j}^{(s)} = N_{i,j}^{(s)}/N$
  $N_{i,j}^{(s)}$: No. of spiking HR neurons in the jth spiking cycle in the ith bursting cycle
- **Pacing degree of spiking times:** $P_{i,j}^{(s)} = \frac{1}{S_{i,j}}\sum_{k=1}^{S_{i,j}} \cos\Phi_k$
  Contribution of spiking times to the macroscopic IPSR R_s
  $S_{i,j}$: Total No. of microscopic spiking times
- **Statistical-Mechanical Spiking Measure** in the ith bursting cycle $M_i^{(s)} = \frac{1}{N_s}\sum_{j=1}^{N_s} M_{i,j}^{(s)}$
- **Statistical-Mechanical Spiking Measure** $M_s = \frac{1}{N_b}\sum_{i=1}^{N_b} M_i^{(s)}$
  With increasing D, $\langle O_s \rangle_r$ decreases very slowly, while $\langle P_s \rangle_r$ and $\langle M_s \rangle_r$ decrease very rapidly.







Summary

- **Characterization of Burst and Spike Synchronizations via Separation of the Slow (Bursting) and Fast (Spiking) Time Scales by Frequency Filtering**
- **Thermodynamic Bursting and Intraburst Spiking Order Parameters**
  - Thermodynamic Bursting Order Parameter → Determination of Threshold for the Bursting Transition
  - Thermodynamic (ntraburst Spiking Order Parameter
    → Determination of Threshold for the Intraburst Spiking Transition
- **Statistical-Mechanical Bursting and Intraburst Spiking Measures**
  Measurement of Degree of Bursting and Intraburst Spiking Synchronizations

References

[1] W. Lim and S.-Y. Kim, "Statistical-mechanical measure of stochastic spiking coherence in a population of inhibitory subthreshold neuron," J. Comput. Neurosci. **31**, 667-677 (2011).
[2] S.-Y. Kim and W. Lim, "Realistic thermodynamic and statistical-mechanical measures for neural synchronization," J. Neurosci. Methods **226**, 161-170 (2014).
[3] S.-Y. Kim and W. Lim, "Thermodynamic order parameters and statistical-mechanical measures for characterization of the burst and spike synchronizations of bursting neurons," e-print: arXiv:1403.3994 [q-bio.NC].
[4] S.-Y. Kim and W. Lim, "Frequency-domain order parameters for the burst and spike synchronization transitions of bursting neurons," e-print: arXiv:1403.4814 [q-bio.NC].